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On one-loop entanglement entropy of two short intervals from OPE of twist operators

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Abstract: We investigate the one-loop entanglement entropy of two short intervals with small cross ratio x on a complex plane in two-dimensional conformal field theory (CFT) using operator product expansion of twist operators. We focus on the one-loop entanglement entropy instead of the general order n Rényi entropy, and this makes the calculation much easier. We consider the contributions of stress tensor to order x^{10}, contributions of W_3 operator to order x^{12}, and contributions of W_4 operator to order x^{14}. The CFT results agree with the ones in gravity.

Keywords: AdS-CFT Correspondence, Conformal and W Symmetry, Field Theories in Lower Dimensions

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Contents

1 Introduction

Entanglement entropy plays an important role in characterizing the correlations of different parts in a many-body system [1, 2]. The usual way of calculating the entanglement entropy is the replica trick [3, 4], in which one firstly calculates the general order n Rényi entropy and then takes the $n \to 1$ limit. It is usually not easy to calculate the entanglement entropy in a quantum field theory, but for a CFT (conformal field theory) that has a gravity dual in AdS (anti-de Sitter) background one can use the AdS/CFT correspondence [5–8] and have a simple calculation. The entanglement entropy of a region A in the boundary CFT is given by the area of a minimal surface $\mathcal{A}$ in the bulk AdS space

$$S_A = \frac{\text{Area}[\mathcal{A}]}{4G}, \tag{1.1}$$

with G being the Newton constant. This is the Ryu-Takayanagi formula of holographic entanglement entropy [9–12]. This is a classical gravity result, and one can also consider the quantum corrections [13–15].

Quantum gravity in AdS_3 spacetime with cosmological constant $\Lambda = -1/\ell^2$ is dual to a two-dimensional CFT with central charge [16]

$$c = \frac{3\ell}{2G}. \tag{1.2}$$

The small Newton constant expansion in gravity side corresponds to large central charge expansion in CFT side [13, 14, 17, 18]. The part of the Rényi entropy that is proportional to central charge is called classical, the next-to-leading part is called one-loop, and the next-to-next-to-leading part is called two-loop, and et. al.

The calculation of N-interval Rényi entropy in a two-dimensional CFT is equivalent to the calculation of a $2N$-point correlation function of twist operators [19]. For one interval on complex plane the Rényi entropy is universal [4, 19], but when for cases of two or more intervals there are no general results and the details of the CFT are relevant [13, 20–23]. For two short intervals on a complex plane, on which we focus in this paper, one can calculate the Rényi entropy as expansion of the cross ratio x in both gravity and CFT sides [13, 14, 23–31]. In CFT side one can use the OPE (operator product expansion) of twist operators, and various cases have been considered [13, 23–30]. Using this method it is very cumbersome to calculate the Rényi entropy to higher order of the cross ratio x. In gravity side the one-loop Rényi entropy of the graviton has been calculated to order x^8 in [14], and this result is reproduced in CFT side by considering contributions of stress tensor in [24, 25]. There is a similar story for the one-interval Rényi entropy on a torus with the temperature being low or high [14, 32–38], but we will not consider the case in this paper.

It was pointed out in [28] that if one takes the $n \to 1$ limit and only calculates the entanglement entropy the calculation would be much easier, both in gravity and in CFT sides. In gravity side, the one-loop entanglement entropy of the graviton has been calculated to order x^{10}, that of the spin-3 field to order x^{14}, and that of the spin-4 field to order x^{18} [28]. In this paper we adopt this strategy and calculate the one-loop entanglement entropy in CFT side. For stress tensor we calculate the one-loop entanglement entropy to order x^{10}, for W_3 operator to order x^{12}, and for W_4 operator to order x^{14}.

The rest of the paper is arranged as follows. In section 2 we review the method of calculating the one-loop two-interval entanglement entropy from OPE of twist operators, as well as the gravity results that we want to reproduce in the CFT side. In section 3 we calculate the contributions of stress tensor to the one-loop two-interval entanglement entropy. In section 4 and 5 we consider the cases W_3 operator and W_4 operator, respectively. We end with conclusion and discussion in section 6. In appendix A there are details of some general calculations that are useful to section 2, 3, and 4. In appendix B there are some summation formulas.

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2 Entanglement entropy from OPE of twist operators

In this section we review small cross ratio expansion of entanglement entropy of two short intervals. We also give the basic setup of the calculation in the paper. It will be very brief here, and one may see details in [13, 23–28].

We consider a two-dimensional CFT on the complex plane, and the constant time slice is an infinite straight line. One can choose a subset A of the line which is the union of several intervals, and name its complement as B. The vacuum state density matrix of the CFT is $\rho = |0\rangle\langle 0|$, and one can trace out the degrees of freedom of B and get the reduced density matrix

$$\rho_A = \mathrm{tr}_B \rho. \tag{2.1}$$

For any positive integer $n > 1$, one can define the order n Rényi entropy

$$S_A^{(n)} = -\frac{1}{n-1} \log \mathrm{tr}_A \rho_A^n. \tag{2.2}$$

For two subsets A and B that do not necessarily complement each other, one may define the Rényi mutual information

$$I_{A,B}^{(n)} = S_A^{(n)} + S_B^{(n)} - S_{A\cup B}^{(n)}. \tag{2.3}$$

Taking the $n \to 1$ limit one gets the entanglement entropy and mutual information.

$$S_A = \lim_{n\to 1} S_A^{(n)}, \quad I_{A,B} = \lim_{n\to 1} I_{A,B}^{(n)}. \tag{2.4}$$

To get the order n Rényi entropy of N intervals, one uses the replica trick and calculates the partition function of the CFT on a genus $(n-1)(N-1)$ Riemann surface. This equals to the correlation function of $2N$ twist operators σ, $\tilde\sigma$ that are inserted at the boundaries of each interval on a complex plane in CFT^n that is the n-fold of the original CFT [19]. The twist operators σ, $\tilde\sigma$ are primary operators with conformal weights [19]

$$h_\sigma = \bar h_\sigma = h_{\tilde\sigma} = \bar h_{\tilde\sigma} = \frac{c(n^2-1)}{24n}. \tag{2.5}$$

For the case of two short intervals in a CFT where all the relevant operators can be decoupled as holomorphic and anti-holomorphic sectors and every anti-holomorphic operator is in one-to-one correspondence with a holomorphic one, one has the Rényi mutual information as a function of the cross ratio x [13, 23–27]

$$I_n = \frac{2}{n-1} \log \left[\sum_K \frac{d_K^2}{\alpha_K} x^{h_K} {}_2F_1(h_K, h_K; 2h_K; x) \right]. \tag{2.6}$$

Here the summation K is over all the holomorphic linearly independent orthogonal quasiprimary operators Φ_K in CFT^n, and every Φ_K is constructed from quasiprimary operators of the original CFT. We call the quasiprimary operators in the original CFT as the old ones, and the quasiprimary operators in CFT^n as the new ones. Factor α_K is the normalization factor of Φ_K

$$\langle \Phi_K(z)\Phi_L(w)\rangle_C = \frac{\alpha_K \delta_{KL}}{(z-w)^{2h_K}}, \tag{2.7}$$

with C denoting the complex plane. Factor d_K is the OPE coefficient and it can be calculated as [23]

$$d_K = \frac{1}{l^{h_K}} \lim_{z\to\infty} z^{2h_K} \langle \Phi_K(z) \rangle_{\mathcal{R}_{n,1}}, \tag{2.8}$$

and here $\mathcal{R}_{n,1}$ is an n-sheeted Riemann surface with the branch cut being $[0,l]$. The expectation value on $\mathcal{R}_{n,1}$ with coordinate z is calculated by mapping it to a complex plane with coordinate f by

$$f(z) = \left(\frac{z-l}{z}\right)^{\frac{1}{n}}. \tag{2.9}$$

When some new quasiprimary operators Φ_{K_p} with $p = 1, 2, \cdots, m$ in CFT^n are not orthogonal to each other, we can either orthogonalize them using Gram-Schmidt process, or just replace the summation of these operators to a product of two vectors and a matrix

$$\frac{d_K^2}{\alpha_K} \to d_K^{\rm T} \alpha_K^{-1} d_K. \tag{2.10}$$

Here $d_K^{\rm T}$ is the transpose of the m-dimensional vector d_K

$$d_K^{\rm T} = (d_{K_1}, d_{K_2}, \cdots, d_{K_m}), \tag{2.11}$$

and α_K is the $m \times m$ normalization matrix

$$\langle \Phi_{K_p} \Phi_{K_q} \rangle_C = \frac{\alpha_{K_{pq}}}{(z-w)^{2h_K}}, \quad p, q = 1, 2, \cdots, m, \tag{2.12}$$

and α_K^{-1} is the inverse of α_K.

To calculate the Rényi mutual information (2.6) to higher order of x, one has to consider a large number of new quasiprimary operators, and this makes the method very cumbersome. However, it was shown in [28] that if one is only interested in the mutual information, i.e. the $n \to 1$ limit of the Rényi mutual information (2.6), the calculation can be simplified significantly. The example of contributions of scalar operators has been given therein. In this paper we will give more examples, including contributions of stress tensor, W_3 operator and W_4 operator. The mutual information is calculated as

$$I = \lim_{n\to 1} \frac{2}{n-1} \left[\sum_K \frac{\hat{d}_K^2}{\alpha_K} x^{h_K} {}_2F_1(h_K, h_K; 2h_K; x) \right], \tag{2.13}$$

with K denoting summation over the nonidentity holomorphic new quasiprimary operators of CFT^n. Here $\hat{d}_K$ is got from d_K by setting all the n's, but the ones in trigonometric functions, to 1. It will not affect the result of mutual information, and it will make the calculation much simpler. We will see in the subsequent sections of this paper that only some of new quasiprimary operators contribute to the mutual information. Furthermore, the central charge c dependence comes from $\hat{d}_K^2/\alpha_K$, and the number of new quasiprimary operators would be smaller if we only want to get the one-loop part of the mutual information.

The method of calculating the one-loop entanglement entropy in the gravity side was developed in [14]. One uses the result in [39–41], and calculate the 1-loop partition function

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in the background of the handlebody.[1] It has been calculated in gravity side that, the spin-2, spin-3, and spin-4 fields contribute to one-loop holographic mutual information

$$
\begin{aligned}
I^{\text{1-loop}}_{\text{spin-2}} &= \frac{x^4}{630}+\frac{2x^5}{693}+\frac{15x^6}{4004}+\frac{x^7}{234}+\frac{167x^8}{36036}+\frac{69422x^9}{14549535}+\frac{122x^{10}}{24871}+O(x^{11}), \\
I^{\text{1-loop}}_{\text{spin-3}} &= \frac{x^6}{12012}+\frac{x^7}{4290}+\frac{7x^8}{16830}+\frac{28x^9}{46189}+\frac{15x^{10}}{19019}+\frac{2x^{11}}{2093}+\frac{1644627x^{12}}{1487285800}+O(x^{13}), \\
I^{\text{1-loop}}_{\text{spin-4}} &= \frac{x^8}{218790}+\frac{4x^9}{230945}+\frac{3x^{10}}{76076}+\frac{5x^{11}}{71162}+\frac{11x^{12}}{101660}+\frac{11x^{13}}{72675}+\frac{1001x^{14}}{5058180}+O(x^{15}).
\end{aligned} \tag{2.14}
$$

In AdS/CFT correspondence, it is standard that the graviton corresponds to stress tensor in CFT side. Also a general spin-s field in gravity side corresponds to W_s and $\bar{W}_s$ operators in CFT side [43, 44]. In this paper we will reproduce the results (2.14) in the CFT side.

3 Stress tensor

In this section we consider an ordinary large central charge CFT, and we get the contributions of vacuum conformal family operators to the one-loop mutual information of two short intervals. The primary operator of the vacuum conformal family is the identity, and the holomorphic decedents are constructed by the stress tensor T, normal ordering and derivatives. Firstly we need to construct the new quasiprimary operators Φ_K we need, then we calculate the coefficients α_K and $\hat{d}_K$, and lastly we sum the results to get the mutual information.

3.1 Construction of quasiprimary operators

For the original CFT, we count the number of vacuum conformal family holomorphic operators in each level as

$$
\chi_{(2)}=\text{tr}_{(2)}x^{L_0}=\prod_{m=2}^{\infty}\frac{1}{1-x^m}=1+x^2+x^3+2x^4+2x^5+4x^6+4x^7+7x^8+8x^9+12x^{10}+O(x^{11}). \tag{3.1}
$$

Then the number of old holomorphic quasiprimary operators in each level is listed as

$$
(1-x)\chi_{(2)}+x=1+x^2+x^4+2x^6+3x^8+x^9+4x^{10}+O(x^{11}). \tag{3.2}
$$

They are listed in table 1. At level 0, it is the identity operator 1. At level 2 we have the stress tensor T and $\alpha_T=\frac{c}{2}$. At level 4 we have

$$
\mathcal{A}=(TT)-\frac{3}{10}\partial^2T, \quad \alpha_{\mathcal{A}}=\frac{c(5c+22)}{10}. \tag{3.3}
$$

At level 6 we have

$$
\begin{aligned}
\mathcal{B} &= (\partial T\partial T)-\frac{4}{5}(T\partial^2T)+\frac{23}{210}\partial^4T, \\
\mathcal{D} &= (T(TT))-\frac{9}{10}(T\partial^2T)+\frac{4}{35}\partial^4T+\frac{93}{70c+29}\mathcal{B}, \\
\alpha_{\mathcal{B}} &= \frac{36c(70c+29)}{175}, \quad \alpha_{\mathcal{D}}=\frac{3c(2c-1)(5c+22)(7c+68)}{4(70c+29)}.
\end{aligned} \tag{3.4}
$$

[1]This gravity result has been recently justified in [42].

level	0	2	4	6	8	9	10	$\cdots$
quasiprimary	1	T	$\mathcal{A}$	$\mathcal{B}$, $\mathcal{D}$	$\mathcal{A}^{(8,m)}$	$\mathcal{A}^{(9)}$	$\mathcal{A}^{(10,m)}$	$\cdots$

Table 1. Old holomorphic quasiprimary operators of vacuum conformal family in the original CFT. The ranges in which the m's take values can be seen easily in (3.2). At level 8 we have $m = 1, 2, 3$, and at level 10 we have $m = 1, 2, 3, 4$.

The quasiprimary operator $\mathcal{D}$ is chosen such that the structure constant $C_{TT\mathcal{D}} = 0$, and $\mathcal{B}$ is chosen such that it is orthogonal to $\mathcal{D}$. At level 8 we have $\mathcal{A}^{(8,m)}$ with $m = 1, 2, 3$, and we need neither their explicit forms or their normalization factors. At level 9 we have $\mathcal{A}^{(9)}$. At level 10 we have $\mathcal{A}^{(10,m)}$ with $m = 1, 2, 3, 4$.

Using the old holomorphic quasiprimary operators of the original CFT listed above as well as derivatives, we can construct all the new holomorphic quasiprimary operators of CFT^n to level 10. Given p old holomorphic quasiprimary operators of original CFT that are located at different replica $\mathcal{O}_{j_1}$, $\mathcal{P}_{j_2}$, $\mathcal{Q}_{j_3}$, $\cdots$, we can just multiply them and get one new quasiprimary operator of CFT^n

$$\mathcal{O}_{j_1}\mathcal{P}_{j_2}\mathcal{Q}_{j_3}\cdots . \tag{3.5}$$

Given also q derivatives, we can get C^q_{p+q-1} linearly independent operators, and so the number of linearly independent quasiprimary operators that can be constructed is

$$C^q_{p+q-1} - C^{q-1}_{p+q-2} = C^q_{p+q-2}. \tag{3.6}$$

We denote these quasiprimary operators with one derivative as

$$\mathrm{I}_m(\mathcal{O}_{j_1}\mathcal{P}_{j_2}\mathcal{Q}_{j_3}\cdots), \quad m = 1, 2, \cdots, p-1, \tag{3.7}$$

or simply

$$\mathrm{I}(\mathcal{O}_{j_1}\mathcal{P}_{j_2}\mathcal{Q}_{j_3}\cdots). \tag{3.8}$$

For all the linearly independent new holomorphic quasiprimary operators with permutations of these j_i's from 0 to $n-1$, we just denote them by

$$\mathrm{I}(\mathcal{O}\mathcal{P}\mathcal{Q}\cdots). \tag{3.9}$$

We use similar notations for the new holomorphic quasiprimary operators of CFT^n with two and more derivatives, and for example we have $\mathbf{II}(\mathcal{O}\mathcal{P}\mathcal{Q}\cdots)$, $\mathbf{III}(\mathcal{O}\mathcal{P}\mathcal{Q}\cdots)$, $\cdots$. We call these operators belong to the class $\mathcal{O}\mathcal{P}\mathcal{Q}\cdots$.

The new holomorphic operators of CFT^n can be counted as $\chi^n_{(2)}$ with $\chi_{(2)}$ being defined in (3.1), and so the new holomorphic quasiprimary operators can be counted as

$$\begin{aligned}(1-x)\chi^n_{(2)}+x = 1+nx^2&+\frac{n(n+1)}{2}x^4+\frac{n(n-1)}{2}x^5+\frac{n(n+1)(n+5)}{6}x^6+\frac{n(n-1)(2n+5)}{6}x^7\\ &+\frac{n(n+1)(n^2+17n+18)}{24}x^8+\frac{n(n+1)(3n^2+19n-10)}{24}x^9\\ &+\frac{n(n+1)(n^3+39n^2+156n+44)}{120}x^{10}+O(x^{11}).\end{aligned} \tag{3.10}$$

We listed all these quasiprimary operators in table 2.

level	quasiprimary	??	#	#
0	1	✓✓	1	1
2	T	× ×	n	n
4	$\mathcal{A}$	× ×	n	$\frac{n(n+1)}{2}$
	TT	✓✓	$\frac{n_2}{2}$	
5	I(TT)	✓✓	$\frac{n_2}{2}$	$\frac{n_2}{2}$
6	$\mathcal{B}$, $\mathcal{D}$	× ×	$2n$	$\frac{n(n+1)(n+5)}{6}$
	$T\mathcal{A}$	× ×	n_2	
	TTT	✓×	$\frac{n_3}{6}$	
	II(TT)	✓✓	$\frac{n_2}{2}$	
7	I$(T\mathcal{A})$	× ×	n_2	$\frac{n(n-1)(2n+5)}{6}$
	I(TTT)	✓×	$\frac{n_3}{3}$	
	III(TT)	✓✓	$\frac{n_2}{2}$	
8	$\mathcal{A}^{(8,m)}$	× ×	$3n$	$\frac{n(n+1)(n^2+17n+18)}{24}$
	$T\mathcal{B}$, $T\mathcal{D}$	× ×	$2n_2$	
	$\mathcal{A}\mathcal{A}$	✓✓	$\frac{n_2}{2}$	
	$TT\mathcal{A}$	✓✓	$\frac{n_3}{2}$	
	$TTTT$	✓✓	$\frac{n_4}{24}$	
	II$(T\mathcal{A})$	× ×	n_2	
	II(TTT)	✓×	$\frac{n_3}{2}$	
	IV(TT)	✓✓	$\frac{n_2}{2}$	
9	$\mathcal{A}^{(9)}$	× ×	n	$\frac{n(n+1)(3n^2+19n-10)}{24}$
	I$(T\mathcal{B})$, I$(T\mathcal{D})$	× ×	$2n_2$	
	I$(\mathcal{A}\mathcal{A})$	✓✓	$\frac{n_2}{2}$	
	I$(TT\mathcal{A})$	✓✓	n_3	
	I$(TTTT)$	✓✓	$\frac{n_4}{8}$	
	III$(T\mathcal{A})$	× ×	n_2	
	III(TTT)	✓×	$\frac{2n_3}{3}$	
	V(TT)	✓✓	$\frac{n_2}{2}$	
10	$\mathcal{A}^{(10,m)}$	× ×	$4n$	$\frac{n(n+1)(n^3+39n^2+156n+44)}{120}$
	$T\mathcal{A}^{(8,m)}$	× ×	$3n_2$	
	$\mathcal{A}\mathcal{B}$, $\mathcal{A}\mathcal{D}$	× ×	$2n_2$	
	$T\mathcal{A}\mathcal{A}$	✓×	$\frac{n_3}{2}$	
	$TT\mathcal{B}$	✓✓	$\frac{n_3}{2}$	
	$TT\mathcal{D}$	× ×	$\frac{n_3}{2}$	
	$TTT\mathcal{A}$	✓×	$\frac{n_4}{6}$	
	$TTTTT$	✓×	$\frac{n_5}{120}$	
	II$(T\mathcal{B})$, **II**$(T\mathcal{D})$	× ×	$2n_2$	
	II$(\mathcal{A}\mathcal{A})$	✓✓	$\frac{n_2}{2}$	
	II$(TT\mathcal{A})$	✓✓	$\frac{3n_3}{2}$	
	II$(TTTT)$	✓✓	$\frac{n_4}{4}$	
	IV$(T\mathcal{A})$	× ×	n_2	
	IV(TTT)	✓×	$\frac{5n_3}{6}$	
	VI(TT)	✓✓	$\frac{n_2}{2}$	
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Table 2. All new holomorphic quasiprimary operators in CFT^n to level 10. If we want to calculate the general order n Rényi mutual information of two short intervals, we have to consider all of them. In the third column we marked the answers to two questions for the operators. The first question is whether the operators contribute to the mutual information, i.e. the order 1 Rényi mutual information, and the second question is whether it contribute to the one-loop part of the mutual information. If one answer is yes, we mark ✓, and if one answer is no, we mark × . In this paper we concentrate on the one-loop mutual information, and so we only need to consider the operators marked with two ✓'s. In the fourth and fifth columns we count the degeneracies, and we have shorthand $n_m = n(n-1)\cdots(n-m+1)$. The counting is in accord with (3.10).

3.2 Calculation of coefficients

If we want to get the general Rényi mutual information using (2.6), we have to get coefficients α_K and d_K for all the operators in table 2. In spirit of [28], after we take $n \to 1$ limit and get the mutual information, only some of them contribute. A general old holomorphic quasiprimary operator $\mathcal{O}$ with conformal weight h transforms in an arbitrary conformal transformation $z \to f(z)$ as

$$\mathcal{O}(z) = f'^h \mathcal{O}(f) + \cdots , \tag{3.11}$$

with $\cdots$ denoting terms that have the Schwarzian derivative or its derivatives. For the transformation (2.9) that we use to calculate d_K, the Schwarzian derivative is proportional

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to $n-1$. We divide the nonidentity new quasiprimary operators of CFT^n in two cases.

- For a new operator with only one nonidentity old quasiprimary operator of the original CFT in one replica, say $\mathcal{O}_j$ with $j = 0, 1, \cdots, n-1$, coefficient d_K only comes from the $\cdots$ in (3.11), and we have $d_K \sim n-1$. So the term $d_K^2/(n-1)$ vanishes in the $n \to 1$ limit. Such operators do not contribute to the mutual information.

- For the other cases, the coefficients d_K is consisted by some trigonometric functions, and terms from $\cdots$ in (3.11) are still proportional to $n-1$. A summation of d_K^2/α_K is just a summation of some trigonometric functions, and this always leads to an overall factor $n-1$. After summation the contributions from $\cdots$ in (3.11) are proportional to $(n-1)^2$, and these terms over $n-1$ would vanish in the $n \to 1$ limit.

From the above analysis, we need not the full form of d_K to get the mutual information, we only need to replace d_K by

$$\hat{d}_K = d_K \text{ by taking all } n \to 1 \text{ except the ones in trigonometric functions.} \tag{3.12}$$

The new coefficient $\hat{d}_K$ is calculated using (2.8), (2.9), (3.12), as well as (3.11) without the $\cdots$.

To make the analysis of the large central charge limit easier, we define the modified normalization factor and the modified OPE coefficient

$$\beta_K = \lim_{c\to\infty} \frac{\alpha_K}{\tilde{\alpha}_K}, \quad \hat{b}_K = \lim_{c\to\infty} \frac{\hat{d}_K}{C_K}, \tag{3.13}$$

with β_K and $\hat{b}_K$ being independent of the central charge. So we have

$$\lim_{c\to\infty} \frac{\hat{d}_K^2}{\alpha_K} = \left(\lim_{c\to\infty} \frac{C_K^2}{\tilde{\alpha}_K} \right) \frac{\hat{b}_K^2}{\beta_K}. \tag{3.14}$$

For CFT^n quasiprimary operators with only one quasiprimary operator of the original CFT, we need not to consider them, as we have said above. For quasiprimary operators with two quasiprimary operators of the original CFT, we only need to consider the cases when the two operators are the same. We have the CFT^n operators of class $\mathcal{O}\mathcal{O}$

$$\mathcal{O}\mathcal{O}, \ \mathrm{I}(\mathcal{O}\mathcal{O}), \ \mathbf{II}(\mathcal{O}\mathcal{O}), \ \cdots. \tag{3.15}$$

For these operators we choose $C_K = \alpha_{\mathcal{O}}$ and $\tilde{\alpha}_K = \alpha_{\mathcal{O}}^2$. For quasiprimary operators with three quasiprimary operators of the original CFT, say class $\mathcal{O}\mathcal{P}\mathcal{Q}$

$$\mathcal{O}\mathcal{P}\mathcal{Q}, \ \mathrm{I}(\mathcal{O}\mathcal{P}\mathcal{Q}), \ \mathbf{II}(\mathcal{O}\mathcal{P}\mathcal{Q}), \ \cdots. \tag{3.16}$$

we choose $C_K = C_{\mathcal{O}\mathcal{P}\mathcal{Q}}$ and $\tilde{\alpha}_K = \alpha_{\mathcal{O}\mathcal{P}\mathcal{Q}} = \alpha_{\mathcal{O}}\alpha_{\mathcal{P}}\alpha_{\mathcal{Q}}$ with $C_{\mathcal{O}\mathcal{P}\mathcal{Q}}$ being the structure constant. For quasiprimary operators with four and more quasiprimary operators of the original CFT, usually we cannot make $\hat{b}_K$ and β_K independent of the central charge, but we can always make them independent of the central charge in the large central charge limit. Coefficients C_K for these cases will be defined case by case. For all the quasiprimary

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operators in class $\mathcal{OPQ}\cdots$, we have the coefficient $\tilde{\alpha}_K = \alpha_{\mathcal{OPQ}\cdots} = \alpha_{\mathcal{O}}\alpha_{\mathcal{P}}\alpha_{\mathcal{Q}}\cdots$. With all these setups, we can easily identify whether some operators contribute to the mutual information or not, and if yes whether they contribute to the one-loop mutual information or not. The answers to the two questions are marked in the third column of table 2. The result is that we only need the quasiprimary operators of the classes TT, $\mathcal{AA}$, $TT\mathcal{A}$, $TTTT$, $TT\mathcal{B}$ to get the one-loop mutual information.

For the classes of TT and $\mathcal{AA}$, the contributions to mutual information are just

$$I_{TT} = I_{\mathcal{OO}}|_{h=2}, \quad I_{\mathcal{AA}} = I_{\mathcal{OO}}|_{h=4}, \tag{3.17}$$

with $I_{\mathcal{OO}}$ being (A.4).

For operators in class of $TT\mathcal{A}$ we have the structure constant

$$C_{TT\mathcal{A}} = \frac{c(5c+22)}{10}. \tag{3.18}$$

To level 10, the quasiprimary operators we need to consider are

$$\begin{aligned}
TT\mathcal{A}, \quad \mathrm{I}_1(TT\mathcal{A}) &= \mathrm{i}\partial TT\mathcal{A} - T\mathrm{i}\partial T\mathcal{A}, \quad \mathrm{I}_2(TT\mathcal{A}) = \mathrm{i}\partial TT\mathcal{A} - \frac{1}{2}TT\mathrm{i}\partial\mathcal{A},\\
\mathbf{II}_1(TT\mathcal{A}) &= \partial T\partial T\mathcal{A} - \frac{2}{5}\partial^2 TT\mathcal{A} - \frac{2}{5}T\partial^2 T\mathcal{A},\\
\mathbf{II}_2(TT\mathcal{A}) &= \partial TT\partial\mathcal{A} - \frac{4}{5}\partial^2 TT\mathcal{A} - \frac{2}{9}TT\partial^2\mathcal{A},\\
\mathbf{II}_3(TT\mathcal{A}) &= T\partial T\partial\mathcal{A} - \frac{4}{5}T\partial^2 T\mathcal{A} - \frac{2}{9}TT\partial^2\mathcal{A}.
\end{aligned} \tag{3.19}$$

We have the modified normalization factors

$$\beta_{TT\mathcal{A}} = 1, \quad \beta_{\mathrm{I}(TT\mathcal{A})} = 2\begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix}, \quad \beta_{\mathbf{II}(TT\mathcal{A})} = \frac{16}{45}\begin{pmatrix} 81 & 36 & 36 \\ 36 & 182 & 20 \\ 36 & 20 & 182 \end{pmatrix}. \tag{3.20}$$

The OPE coefficients are

$$\begin{aligned}
\hat{b}^{j_1j_2j_3}_{TT\mathcal{A}} &= \frac{1}{2^8}\frac{1}{s^4_{j_1j_3}s^4_{j_2j_3}}, \qquad \hat{b}^{j_1j_2j_3}_{\mathrm{I}_1(TT\mathcal{A})} = \frac{1}{2^7}\frac{c_{j_1j_2}}{s^5_{j_1j_3}s^5_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathrm{I}_2(TT\mathcal{A})} &= \frac{1}{2^8}\frac{s_{j_1j_2}-2s_{j_1j_2j_3}}{s^5_{j_1j_3}s^5_{j_2j_3}}, \quad \hat{b}^{j_1j_2j_3}_{\mathbf{II}_1(TT\mathcal{A})} = -\frac{1}{2^7}\frac{s^2_{j_1j_2}}{s^6_{j_1j_3}s^6_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_2(TT\mathcal{A})} &= \frac{1}{9\cdot 2^8}\frac{10s^2_{j_1j_2}-36s^2_{j_1j_3}+45s^2_{j_2j_3}+36s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^6_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_3(TT\mathcal{A})} &= \frac{1}{9\cdot 2^8}\frac{10s^2_{j_1j_2}+45s^2_{j_1j_3}-36s^2_{j_2j_3}+36s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^6_{j_2j_3}},
\end{aligned} \tag{3.21}$$

with the definitions $s_{j_1j_2} = \sin(\frac{j_1-j_2}{n}\pi)$, $s_{j_1j_2j_3} = \sin(\frac{j_1+j_2-2j_3}{n}\pi)$, $c_{j_1j_2} = \cos(\frac{j_1-j_2}{n}\pi)$ and the ones similar to them.

For operators in class $TTTT$, we choose

$$C_K = \frac{c^2}{4}. \tag{3.22}$$

To level 10, we need the operators

$$
\begin{aligned}
TTTT,\quad \mathrm{I}_1(TTTT) &= \mathrm{i}\partial TTTT - T\mathrm{i}\partial TTT,\\
\mathrm{I}_2(TTTT) &= \mathrm{i}\partial TTTT - TT\mathrm{i}\partial TT,\quad \mathrm{I}_3(TTTT) = \mathrm{i}\partial TTTT - TTT\mathrm{i}\partial T,\\
\mathbf{II}_1(TTTT) &= \partial T\partial TTT - \frac{2}{5}\partial^2 TTTT - \frac{2}{5}T\partial^2 TTT,\\
\mathbf{II}_2(TTTT) &= \partial TT\partial TT - \frac{2}{5}\partial^2 TTTT - \frac{2}{5}TT\partial^2 TT,\\
\mathbf{II}_3(TTTT) &= \partial TTT\partial T - \frac{2}{5}\partial^2 TTTT - \frac{2}{5}TTT\partial^2 T,\\
\mathbf{II}_4(TTTT) &= T\partial T\partial TT - \frac{2}{5}T\partial^2 TTT - \frac{2}{5}TT\partial^2 TT,\\
\mathbf{II}_5(TTTT) &= T\partial TT\partial T - \frac{2}{5}T\partial^2 TTT - \frac{2}{5}TTT\partial^2 T,\\
\mathbf{II}_6(TTTT) &= TT\partial T\partial T - \frac{2}{5}TT\partial^2 TT - \frac{2}{5}TTT\partial^2 T.
\end{aligned}
\tag{3.23}
$$

The modified normalization factors are

$$
\beta_{TTTT}=1,\quad \beta_{\mathrm{I}(TTTT)}=4\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix},\quad \beta_{\mathbf{II}(TTTT)}=\frac{16}{5}\begin{pmatrix}9&2&2&2&2&0\\2&9&2&2&0&2\\2&2&9&0&2&2\\2&2&0&9&2&2\\2&0&2&2&9&2\\0&2&2&2&2&9\end{pmatrix}. \tag{3.24}
$$

We need the leading part of the four-point function

$$
\langle T(f_1)T(f_2)T(f_3)T(f_4)\rangle_C = \frac{c^2}{4}\left(\frac{1}{f_{12}^4 f_{34}^4}+\frac{1}{f_{13}^4 f_{24}^4}+\frac{1}{f_{14}^4 f_{23}^4}\right)+O(c), \tag{3.25}
$$

with the definition $f_{12}=f_1-f_2$ and the ones similar to it. The modified OPE coefficients are

$$
\begin{aligned}
\hat{b}_{TTTT}^{j_1j_2j_3j_4} &= \frac{1}{2^8}\left(\frac{1}{s_{j_1j_2}^4 s_{j_3j_4}^4}+\frac{1}{s_{j_1j_3}^4 s_{j_2j_4}^4}+\frac{1}{s_{j_1j_4}^4 s_{j_2j_3}^4}\right),\\
\hat{b}_{\mathrm{I}_1(TTTT)}^{j_1j_2j_3j_4} &= \frac{1}{2^7}\left(\frac{s_{j_1j_3j_2j_4}}{s_{j_1j_3}^5 s_{j_2j_4}^5}+\frac{s_{j_1j_4j_2j_3}}{s_{j_1j_4}^5 s_{j_2j_3}^5}-\frac{2c_{j_1j_2}}{s_{j_1j_2}^5 s_{j_3j_4}^4}\right),\\
\hat{b}_{\mathrm{I}_2(TTTT)}^{j_1j_2j_3j_4} &= \hat{b}_{\mathrm{I}_1(TTTT)}^{j_1j_3j_2j_4},\qquad \hat{b}_{\mathrm{I}_3(TTTT)}^{j_1j_2j_3j_4} = \hat{b}_{\mathrm{I}_1(TTTT)}^{j_1j_4j_2j_3},\\
\hat{b}_{\mathbf{II}_1(TTTT)}^{j_1j_2j_3j_4} &= \frac{1}{2^7}\left(\frac{s_{j_1j_3j_2j_4}^2}{s_{j_1j_3}^6 s_{j_2j_4}^6}+\frac{s_{j_1j_4j_2j_3}^2}{s_{j_1j_4}^6 s_{j_2j_3}^6}+\frac{9-8s_{j_1j_2}^2}{2s_{j_1j_2}^6 s_{j_3j_4}^4}\right),\\
\hat{b}_{\mathbf{II}_2(TTTT)}^{j_1j_2j_3j_4} &= \hat{b}_{\mathbf{II}_1(TTTT)}^{j_1j_3j_2j_4},\qquad \hat{b}_{\mathbf{II}_3(TTTT)}^{j_1j_2j_3j_4} = \hat{b}_{\mathbf{II}_1(TTTT)}^{j_1j_4j_2j_3},\\
\hat{b}_{\mathbf{II}_4(TTTT)}^{j_1j_2j_3j_4} &= \hat{b}_{\mathbf{II}_1(TTTT)}^{j_2j_3j_1j_4},\qquad \hat{b}_{\mathbf{II}_5(TTTT)}^{j_1j_2j_3j_4} = \hat{b}_{\mathbf{II}_1(TTTT)}^{j_2j_4j_1j_3},\qquad \hat{b}_{\mathbf{II}_6(TTTT)}^{j_1j_2j_3j_4} = \hat{b}_{\mathbf{II}_1(TTTT)}^{j_3j_4j_1j_2}.
\end{aligned}
\tag{3.26}
$$

Here there are new definition $s_{j_1j_3j_2j_4}=\sin(\frac{j_1-j_3-j_2+j_4}{n}\pi)$ and the ones similar to it.

For operators in class $TT\mathcal{B}$, we have the structure constant

$$
C_{TT\mathcal{B}} = -\frac{2c(70c+29)}{35}, \tag{3.27}
$$

and the operators, modified normalization factors, and modified OPE coefficients are

$$TT\mathcal{B}, \quad \beta_{TT\mathcal{B}} = 1, \quad \hat{b}^{j_1 j_2 j_3}_{TT\mathcal{B}} = -\frac{1}{2^{10}} \frac{s^2_{j_1 j_2}}{s^6_{j_1 j_3} s^6_{j_2 j_3}}. \tag{3.28}$$

3.3 One-loop mutual information

Using the coefficients in the last subsection and the summation formulas in appendix B we can get the one-loop mutual information. The contributions from operators of class TT, $\mathcal{AA}$, $TT\mathcal{A}$, $TTTT$, and $TT\mathcal{B}$ are respectively

$$\begin{aligned}
I^{\text{1-loop}}_{TT} &= \frac{x^4}{630} + \frac{2x^5}{693} + \frac{15x^6}{4004} + \frac{x^7}{234} + \frac{7x^8}{1530} + \frac{84x^9}{17765} + \frac{x^{10}}{209} + O(x^{11}), \\
I^{\text{1-loop}}_{\mathcal{AA}} &= \frac{x^8}{218790} + \frac{4x^9}{230945} + \frac{3x^{10}}{76076} + O(x^{11}), \\
I^{\text{1-loop}}_{TT\mathcal{A}} &= -\frac{x^8}{109395} - \frac{8x^9}{230945} - \frac{3x^{10}}{38038} + O(x^{11}), \\
I^{\text{1-loop}}_{TTTT} &= \frac{x^8}{15708} + \frac{878x^9}{14549535} + \frac{207x^{10}}{1293292} + O(x^{11}), \quad I^{\text{1-loop}}_{TT\mathcal{B}} = O(x^{11}).
\end{aligned} \tag{3.29}$$

Summing them together, we get the contributions of the vacuum conformal family to one-loop mutual information

$$I^{\text{1-loop}}_{(2)} = \frac{x^4}{630} + \frac{2x^5}{693} + \frac{15x^6}{4004} + \frac{x^7}{234} + \frac{167x^8}{36036} + \frac{69422x^9}{14549535} + \frac{122x^{10}}{24871} + O(x^{11}), \tag{3.30}$$

and this matches the gravity result in [28], i.e. $I^{\text{1-loop}}_{\text{spin-2}}$ in (2.14). Note that $I^{\text{1-loop}}_{TT}$ matches $I^{\text{1-loop}}_{(2)}$ to order x^7.

4 W_3 operator

In a CFT with $W(2,3)$ symmetry, there are operators W with conformal weights (3,0) and $\bar{W}$ with conformal weights (0,3) besides the operators T and $\bar{T}$. In such a CFT the contributions from the stress tensor still exist. In this section we consider the additional contributions to the one-loop mutual information because of the existence of the W operator.

4.1 Construction of quasiprimary operators

We count the holographic operators in the original CFT with $W(2,3)$ symmetry as

$$\chi_{(2,3)} = \text{tr}_{(2,3)} x^{L_0} = \prod_{m=0}^{\infty} \frac{1}{1-x^{m+2}} \frac{1}{1-x^{m+3}}. \tag{4.1}$$

The holomorphic quasiprimary operators are counted as

$$(1-x)\chi_{(2,3)} + x = 1 + x^2 + x^3 + x^4 + x^5 + 4x^6 + 2x^7 + 7x^8 + 7x^9 + 12x^{10} + 14x^{11} + 26x^{12} + O(x^{13}), \tag{4.2}$$

and the additional ones compared to an ordinary CFT are counted as

$$(1-x)(\chi_{(2,3)} - \chi_{(2)}) = x^3 + x^5 + 2x^6 + 2x^7 + 4x^8 + 6x^9 + 8x^{10} + 12x^{11} + 19x^{12} + O(x^{13}), \tag{4.3}$$

with $\chi_{(2)}$ being defined in (3.1). The holomorphic operators in the conformal family of a general holomorphic nonidentity primary operator ϕ with conformal weights $(h, 0)$ are counted as

$$\chi_\phi = \mathrm{tr}_\phi x^{L_0} = x^h \chi, \quad \chi \equiv \prod_{m=1}^{\infty} \frac{1}{1-x^m}. \tag{4.4}$$

The number of quasiprimary operators in conformal family of ϕ is counted as

$$(1-x)\chi_\phi = x^h \left[1+x^2+x^3+2x^4+2x^5+4x^6+4x^7+7x^8+8x^9+12x^{10}+O(x^{11})\right]. \tag{4.5}$$

When ϕ is the operator W we have $h=3$, and we choose $\alpha_W = \frac{c}{3}$ as usual. At level 5, we have the quasiprimary operator

$$\mathcal{U} = (TW) - \frac{3}{14}\partial^2 W, \quad \alpha_\mathcal{U} = \frac{c(7c+114)}{42}. \tag{4.6}$$

At level 6, we have

$$\mathcal{V} = (T\mathrm{i}\partial W) - \frac{3}{2}(\mathrm{i}\partial TW) - \frac{1}{8}\mathrm{i}\partial^3 W, \quad \alpha_\mathcal{V} = \frac{5c(c+2)}{2}. \tag{4.7}$$

At level 7, we have two quasiprimary operators

$$\begin{aligned}
\mathcal{X} &= (\partial T\partial W) - \frac{2}{7}(T\partial^2 W) - \frac{3}{5}(\partial^2 TW) + \frac{1}{42}\partial^4 W, \\
\mathcal{Z} &= (T(TW)) - \frac{3}{7}(T\partial^2 W) - \frac{3}{10}(\partial^2 TW) + \frac{1}{28}\partial^4 W + \frac{141}{35c+53}\mathcal{X}, \\
\alpha_\mathcal{X} &= \frac{264c(35c+53)}{1225}, \quad \alpha_\mathcal{Z} = \frac{c(c+23)(5c-4)(7c+114)}{6(35c+53)}.
\end{aligned} \tag{4.8}$$

Here $\mathcal{Z}$ is chosen such that the structure constant $C_{TW\mathcal{Z}} = 0$, and $\mathcal{X}$ is chosen such that it is orthogonal to $\mathcal{Z}$. We also have the useful structure constants

$$C_{TW\mathcal{U}} = \frac{c(7c+114)}{42}, \quad C_{TW\mathcal{V}} = -\mathrm{i}c(c+2), \quad C_{TW\mathcal{X}} = -\frac{2c(35c+53)}{35}. \tag{4.9}$$

The additional holomorphic primary operators in the original CFT with $W(2,3)$ symmetry are counted as

$$\frac{\chi_{(2,3)} - \chi_{(2)}}{\chi} = x^3 + x^6 + x^8 + x^9 + x^{10} + x^{11} + 3x^{12} + O(x^{13}), \tag{4.10}$$

with $\chi_{(2)}$ in (3.1), $\chi_{(2,3)}$ in (4.1), and χ in (4.4). At level 3, it is just W, and at level 6, 8, 9, 10, 11 we name them $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$, respectively. At level 12, there are three of them, and we name them $\mathcal{J}$, $\mathcal{K}$, $\mathcal{L}$. We list them and their decedent quasiprimary operators in table 3. The explicit form of $\mathcal{E}$ can be found in, for example, the review [45], from which we can get

$$\alpha_\mathcal{E} = \frac{2c^2}{9} + O(c), \quad C_{WW\mathcal{E}} = \frac{2c^2}{9} + O(c). \tag{4.11}$$

The explicit forms, normalization factors, structure constants of other primary operators will not be used in this paper.

L_0	2	3	4	5	6	7	8	9	10	11	12	$\cdots$
#	1	1	1	1	4	2	7	7	12	14	26	$\cdots$
1	T		$\mathcal{A}$		$\mathcal{A}^{(6,m)}$		$\mathcal{A}^{(8,m)}$	$\mathcal{A}^{(9)}$	$\mathcal{A}^{(10,m)}$	$\mathcal{A}^{(11,m)}$	$\mathcal{A}^{(12,m)}$	$\cdots$
W		W		$\mathcal{U}$	$\mathcal{V}$	$\mathcal{X}$, $\mathcal{Z}$	$W^{(8,m)}$	$W^{(9,m)}$	$W^{(10,m)}$	$W^{(11,m)}$	$W^{(12,m)}$	$\cdots$
$\mathcal{E}$					$\mathcal{E}$		$\mathcal{E}^{(8)}$	$\mathcal{E}^{(9)}$	$\mathcal{E}^{(10,m)}$	$\mathcal{E}^{(11,m)}$	$\mathcal{E}^{(12,m)}$	$\cdots$
$\mathcal{F}$							$\mathcal{F}$		$\mathcal{F}^{(10)}$	$\mathcal{F}^{(11)}$	$\mathcal{F}^{(12,m)}$	$\cdots$
$\mathcal{G}$								$\mathcal{G}$		$\mathcal{G}^{(11)}$	$\mathcal{G}^{(12)}$	$\cdots$
$\mathcal{H}$									$\mathcal{H}$		$\mathcal{H}^{(12)}$	$\cdots$
$\mathcal{I}$										$\mathcal{I}$		$\cdots$
$\mathcal{J}$, $\mathcal{K}$, $\mathcal{L}$											$\mathcal{J}$, $\mathcal{K}$, $\mathcal{L}$	$\cdots$

Table 3. Old holographic nonidentity quasiprimary operators in the original CFT with $W(2,3)$ symmetry. In the first line, there are the levels. In second line it is the number of quasiprimary operators in each level. From the third line, we list the quasiprimary operators in each conformal family, and the primary operator for each conformal family is given at the first column. There are some m's in the table, and they take values in different ranges. The range that each m takes values can be figured in (3.2) and (4.5).

The additional new holomorphic quasiprimary operators in CFT^n with $W(2,3)$ symmetry compared with an ordinary CFT^n, are counted as

$$\begin{aligned}(1-x)(\chi^n_{(2,3)}-\chi^n_{(2)}) = nx^3+n^2x^5+\frac{n(3n+1)}{2}x^6+\frac{n(n^2+4n-1)}{2}x^7+\frac{n(n+1)(3n+1)}{2}x^8\\ +\frac{n(n+1)(n^2+18n-1)}{6}x^9+\frac{n(9n^3+58n^2+27n+2)}{12}x^{10}\\ +\frac{n(n^4+52n^3+179n^2+68n-12)}{24}x^{11}\\ +\frac{n(6n^4+109n^3+232n^2+83n+26)}{24}x^{12}+O(x^{13}).\end{aligned} \tag{4.12}$$

They are listed in table 4.

4.2 Calculation of coefficients

To level 12, the new holomorphic quasiprimary operators in CFT^n that contribute to the one-loop mutual information are the ones in classes WW, $\mathcal{UU}$, $TW\mathcal{U}$, $TTWW$, $TW\mathcal{V}$, $\mathcal{VV}$, $\mathcal{EE}$, $TW\mathcal{X}$, $WW\mathcal{E}$, and $WWWW$. The contributions of operators in classes WW, $\mathcal{UU}$, $\mathcal{VV}$, and $\mathcal{EE}$ are

$$I_{WW}=I_{\mathcal{OO}}|_{h=3},\quad I_{\mathcal{UU}}=I_{\mathcal{OO}}|_{h=5},\quad I_{\mathcal{VV}}=I_{\mathcal{EE}}=I_{\mathcal{OO}}|_{h=6}, \tag{4.13}$$

with $I_{\mathcal{OO}}$ in (A.4).

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L_0	quasiprimary	???	#	#
3	W	$\times\times\times$	n	n
5	$\mathcal{U}$	$\times\times\times$	n	n^2
	TW	$\times\times\times$	n_2	
6	$\mathcal{V}$, $\mathcal{E}$	$\times\times\times$	$2n$	$\frac{n(3n+1)}{2}$
	WW	$\checkmark\checkmark\checkmark$	$\frac{n_2}{2}$	
	n_2			
7	$\mathcal{X}$, $\mathcal{Z}$	$\times\times\times$	$2n$	$\frac{n(n^2+4n-1)}{2}$
	$T\mathcal{U}$, $\mathcal{A}W$	$\times\times\times$	$2n_2$	
	TTW	$\times\times\times$	$\frac{n_3}{2}$	
	$\frac{3n_2}{2}$			
8	$W^{(8,m)}$, $\mathcal{E}^{(8)}$, $\mathcal{F}$	$\times\times\times$	$4n$	$\frac{n(n+1)(3n+1)}{2}$
	$T\mathcal{V}$, $T\mathcal{E}$	$\times\times\times$	$2n_2$	
	$W\mathcal{U}$	$\checkmark\times\times$	n_2	
	TWW	$\checkmark\checkmark\times$	$\frac{n_3}{2}$	
	$\frac{n(n-1)(2n+3)}{2}$			
9	$W^{(9,m)}$, $\mathcal{E}^{(9)}$, $\mathcal{G}$	$\times\times\times$	$6n$	$\frac{n(n+1)(n^2+18n-1)}{6}$
	$T\mathcal{X}$, $T\mathcal{Z}$, $\mathcal{AU}$, $\mathcal{A}^{(6,m)}W$	$\times\times\times$	$5n_2$	
	$W\mathcal{V}$	$\checkmark\times\times$	n_2	
	$W\mathcal{E}$	$\times\times\times$	n_2	
	$TT\mathcal{U}$, $T\mathcal{A}W$, WWW	$\times\times\times$	$\frac{5n_3}{3}$	
	$TTTW$	$\times\times\times$	$\frac{n_4}{6}$	
	$\frac{n(n-1)(5n+3)}{2}$			
10	$W^{(10,m)}$, $\mathcal{E}^{(10,m)}$, $\mathcal{F}^{(10)}$, $\mathcal{H}$	$\times\times\times$	$8n$	$\frac{n(9n^3+58n^2+27n+2)}{12}$
	$TW^{(8,m)}$, $T\mathcal{E}^{(8)}$, $T\mathcal{F}$, $\mathcal{AV}$, $\mathcal{AE}$	$\times\times\times$	$6n_2$	
	$W\mathcal{X}$, $W\mathcal{Z}$	$\checkmark\times\times$	$2n_2$	
	$\mathcal{UU}$	$\checkmark\checkmark\checkmark$	$\frac{n_2}{2}$	
	$TT\mathcal{V}$, $TT\mathcal{E}$	$\times\times\times$	n_3	
	$TW\mathcal{U}$	$\checkmark\checkmark\checkmark$	n_3	
	$\mathcal{A}WW$	$\checkmark\checkmark\times$	$\frac{n_3}{2}$	
	$TTWW$	$\checkmark\checkmark\checkmark$	$\frac{n_4}{4}$	
	$\frac{n(n-1)(3n^2+26n+17)}{6}$			
11	$W^{(11,m)}$, $\mathcal{E}^{(11,m)}$, $\mathcal{F}^{(11)}$, $\mathcal{G}^{(11)}$, $\mathcal{I}$	$\times\times\times$	$12n$	$\frac{n(n^4+52n^3+179n^2+68n-12)}{24}$
	$TW^{(9,m)}$, $T\mathcal{E}^{(9)}$, $T\mathcal{G}$, $\mathcal{AX}$, $\mathcal{AZ}$, $\mathcal{A}^{(6,m)}\mathcal{U}$, $\mathcal{A}^{(8,m)}W$	$\times\times\times$	$13n_2$	
	$WW^{(8,m)}$, $\mathcal{UV}$	$\checkmark\times\times$	$3n_2$	
	$W\mathcal{E}^{(8)}$, $W\mathcal{F}$, $\mathcal{UE}$	$\times\times\times$	$3n_2$	
	$TT\mathcal{X}$, $TT\mathcal{Z}$, $T\mathcal{AU}$, $T\mathcal{A}^{(6,m)}W$, $\mathcal{AA}W$	$\times\times\times$	$\frac{9n_3}{2}$	
	$TW\mathcal{V}$	$\checkmark\checkmark\checkmark$	n_3	
	$TW\mathcal{E}$, $WW\mathcal{U}$	$\times\times\times$	$\frac{n_3}{2}$	
	$TTT\mathcal{U}$, $TT\mathcal{A}W$	$\times\times\times$	$\frac{2n_4}{3}$	
	$TWWW$	$\times\times\times$	$\frac{n_4}{6}$	
	$TTTTW$	$\times\times\times$	$\frac{n_5}{24}$	
	$\frac{n(n-1)(9n^2+67n+94)}{12}$			
12	$W^{(12,m)}$, $\mathcal{E}^{(12,m)}$, $\mathcal{F}^{(12,m)}$, $\mathcal{G}^{(12)}$, $\mathcal{H}^{(12)}$, $\mathcal{J}$, $\mathcal{K}$, $\mathcal{L}$	$\times\times\times$	$19n$	$\frac{n(6n^4+109n^3+232n^2+83n+26)}{24}$
	$TW^{(10,m)}$, $T\mathcal{E}^{(10,m)}$, $T\mathcal{F}^{(10)}$, $T\mathcal{H}$, $\mathcal{A}W^{(8,m)}$, $\mathcal{AE}^{(8)}$, $\mathcal{AF}$, $\mathcal{A}^{(6,m)}\mathcal{V}$, $\mathcal{A}^{(6,m)}\mathcal{E}$, $\mathcal{A}^{(9)}W$	$\times\times\times$	$17n_2$	
	$WW^{(9,m)}$, $\mathcal{UX}$, $\mathcal{UZ}$	$\checkmark\times\times$	$6n_2$	
	$W\mathcal{E}^{(9)}$, $W\mathcal{G}$, $\mathcal{VE}$	$\times\times\times$	$3n_2$	
	$\mathcal{VV}$, $\mathcal{EE}$	$\checkmark\checkmark\checkmark$	n_2	
	$TTW^{(8,m)}$, $TT\mathcal{E}^{(8)}$, $TT\mathcal{F}$, $T\mathcal{AV}$, $T\mathcal{AE}$	$\times\times\times$	$4n_3$	
	$\mathcal{A}^{(6,m)}WW$, $\mathcal{A}W\mathcal{U}$, $T\mathcal{UU}$	$\checkmark\checkmark\times$	$\frac{5n_3}{2}$	
	$TW\mathcal{X}$	$\checkmark\checkmark\checkmark$	n_3	
	$TW\mathcal{Z}$, $WW\mathcal{V}$	$\times\times\times$	$\frac{3n_3}{2}$	
	$WW\mathcal{E}$	$\checkmark\checkmark\checkmark$	$\frac{n_3}{2}$	
	$TTT\mathcal{V}$, $TTT\mathcal{E}$	$\times\times\times$	$\frac{n_4}{3}$	
	$TTW\mathcal{U}$, $T\mathcal{A}WW$	$\checkmark\checkmark\times$	n_4	
	$WWWW$	$\checkmark\checkmark\checkmark$	$\frac{n_4}{24}$	
	$TTTWW$	$\checkmark\checkmark\times$	$\frac{n_5}{12}$	
	$\frac{n(n-1)(n^3+53n^2+232n+300)}{24}$			
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Table 4. Additional new holographic quasiprimary operators in CFT^n with $W(2,3)$ symmetry. The operators with derivatives can be constructed from the ones without derivatives easily, and so we only list the number of such operators in each level. In the third column we mark whether the operators contribute to the Rényi mutual information I_n, mutual information I, and one-loop part of mutual information $I_{\text{1-loop}}$. The counting in the fourth and fifth columns is in accord with (4.12).

For operators in class $TW\mathcal{U}$, we have

$$\begin{aligned}
TW\mathcal{U},\quad I_1(TW\mathcal{U}) &= \mathrm{i}\partial TW\mathcal{U} - \frac{2}{3}T\mathrm{i}\partial W\mathcal{U},\quad I_2(TW\mathcal{U}) = \mathrm{i}\partial TW\mathcal{U} - \frac{2}{5}TW\mathrm{i}\partial\mathcal{U},\\
\mathbf{II}_1(TW\mathcal{U}) &= \partial T\partial W\mathcal{U} - \frac{3}{5}\partial^2 TW\mathcal{U} - \frac{2}{7}T\partial^2 W\mathcal{U},\\
\mathbf{II}_2(TW\mathcal{U}) &= \partial TW\partial\mathcal{U} - \partial^2 TW\mathcal{U} - \frac{2}{11}TW\partial^2\mathcal{U},\\
\mathbf{II}_3(TW\mathcal{U}) &= T\partial W\partial\mathcal{U} - \frac{5}{7}T\partial^2 W\mathcal{U} - \frac{3}{11}TW\partial^2\mathcal{U},
\end{aligned} \tag{4.14}$$

the modified normalization factors

$$\beta_{TW\mathcal{U}} = 1,\quad \beta_{\mathrm{I}(TW\mathcal{U})} = \frac{4}{15}\begin{pmatrix} 25 & 15\\ 15 & 21\end{pmatrix},\quad \beta_{\mathbf{II}(TW\mathcal{U})} = \frac{12}{385}\begin{pmatrix} 1452 & 770 & 550\\ 770 & 2800 & 350\\ 550 & 350 & 3825\end{pmatrix}, \tag{4.15}$$

and the modified OPE coefficients

$$\begin{aligned}
\hat{b}^{j_1j_2j_3}_{TW\mathcal{U}} &= -\frac{1}{2^{10}}\frac{1}{s^4_{j_1j_3}s^6_{j_2j_3}}, & \hat{b}^{j_1j_2j_3}_{\mathrm{I}_1(TW\mathcal{U})} &= -\frac{1}{2^9}\frac{s_{j_1j_2}}{s^5_{j_1j_3}s^7_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathrm{I}_2(TW\mathcal{U})} &= -\frac{1}{5\cdot 2^9}\frac{2s_{j_1j_2}-5s_{j_1j_2j_3}}{s^5_{j_1j_3}s^7_{j_2j_3}}, & \hat{b}^{j_1j_2j_3}_{\mathbf{II}_1(TW\mathcal{U})} &= -\frac{3}{2^{10}}\frac{s^2_{j_1j_2}}{s^6_{j_1j_3}s^8_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_2(TW\mathcal{U})} &= -\frac{1}{11\cdot 2^{10}}\frac{10s^2_{j_1j_2}-44s^2_{j_1j_3}+55s^2_{j_2j_3}+55s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^8_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_3(TW\mathcal{U})} &= -\frac{3}{11\cdot 2^{11}}\frac{21s^2_{j_1j_2}+77s^2_{j_1j_3}-66s^2_{j_2j_3}+55s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^8_{j_2j_3}}.
\end{aligned} \tag{4.16}$$

In class $TW\mathcal{V}$, we have operators

$$TW\mathcal{V},\quad \mathrm{I}_1(TW\mathcal{V}) = \mathrm{i}\partial TW\mathcal{V} - \frac{2}{3}T\mathrm{i}\partial W\mathcal{V},\quad \mathrm{I}_2(TW\mathcal{V}) = \mathrm{i}\partial TW\mathcal{V} - \frac{1}{3}TW\mathrm{i}\partial\mathcal{V}, \tag{4.17}$$

the modified normalization factors

$$\beta_{TW\mathcal{V}} = 1,\quad \beta_{\mathrm{I}(TW\mathcal{V})} = \frac{4}{3}\begin{pmatrix} 5 & 3\\ 3 & 4\end{pmatrix}, \tag{4.18}$$

and the modified OPE coefficients

$$\begin{aligned}
\hat{b}^{j_1j_2j_3}_{TW\mathcal{V}} &= \frac{\mathrm{i}}{2^{11}}\frac{s_{j_1j_2}}{s^5_{j_1j_3}s^7_{j_2j_3}}, \quad \hat{b}^{j_1j_2j_3}_{\mathrm{I}_1(TW\mathcal{V})} = \frac{\mathrm{i}}{3\cdot 2^{12}}\frac{12s^2_{j_1j_2}+2s^2_{j_1j_3}+3s^2_{j_2j_3}}{s^6_{j_1j_3}s^8_{j_2j_3}},\\
\hat{b}^{j_1j_2j_3}_{\mathrm{I}_2(TW\mathcal{V})} &= \frac{\mathrm{i}}{3\cdot 2^{12}}\frac{5s^2_{j_1j_2}-12s^2_{j_1j_3}+15s^2_{j_2j_3}}{s^6_{j_1j_3}s^8_{j_2j_3}}.
\end{aligned} \tag{4.19}$$

For operators in class $TW\mathcal{X}$, we have

$$TW\mathcal{X},\quad \beta_{TW\mathcal{X}} = 1,\quad \hat{b}^{j_1j_2j_3}_{TW\mathcal{X}} = \frac{1}{2^{12}}\frac{s^2_{j_1j_2}}{s^6_{j_1j_3}s^8_{j_2j_3}}. \tag{4.20}$$

For operators in class $WW\mathcal{E}$, we have

$$WW\mathcal{E}, \quad \beta_{WW\mathcal{E}} = 1, \quad \hat{b}^{j_1j_2j_3}_{WW\mathcal{E}} = \frac{1}{2^{12}} \frac{1}{s^6_{j_1j_3} s^6_{j_2j_3}}. \tag{4.21}$$

In class $TTWW$, we choose $C_K = \frac{c^2}{6}$ and we have operators

$$\begin{aligned}
TTWW, \quad \mathrm{I}_1(TTWW) &= \mathrm{i}\partial TTWW - T\mathrm{i}\partial TWW, \\
\mathrm{I}_2(TTWW) &= \mathrm{i}\partial TTWW - \frac{2}{3} TT\mathrm{i}\partial WW, \\
\mathrm{I}_3(TTWW) &= \mathrm{i}\partial TTWW - \frac{2}{3} TTW\mathrm{i}\partial W, \\
\mathbf{II}_1(TTWW) &= \partial T\partial TWW - \frac{2}{5}\partial^2 TTWW - \frac{2}{5} T\partial^2 TWW, \\
\mathbf{II}_2(TTWW) &= \partial TT\partial WW - \frac{3}{5}\partial^2 TTWW - \frac{2}{7} TT\partial^2 WW, \\
\mathbf{II}_3(TTWW) &= \partial TTW\partial W - \frac{3}{5}\partial^2 TTWW - \frac{2}{7} TTW\partial^2 W, \\
\mathbf{II}_4(TTWW) &= T\partial T\partial WW - \frac{3}{5} T\partial^2 TWW - \frac{2}{7} TT\partial^2 WW, \\
\mathbf{II}_5(TTWW) &= T\partial TW\partial W - \frac{3}{5} T\partial^2 TWW - \frac{2}{7} TTW\partial^2 W, \\
\mathbf{II}_6(TTWW) &= TT\partial W\partial W - \frac{3}{7} TT\partial^2 WW - \frac{3}{7} TTW\partial^2 W,
\end{aligned} \tag{4.22}$$

modified normalization factors

$$\beta_{TTWW} = 1, \quad \beta_{\mathrm{I}(TTWW)} = \frac{4}{3}\begin{pmatrix} 6 & 3 & 3 \\ 3 & 5 & 3 \\ 3 & 3 & 5 \end{pmatrix}, \quad \beta_{\mathbf{II}(TTWW)} = \frac{12}{35}\begin{pmatrix} 84 & 28 & 28 & 28 & 28 & 0 \\ 28 & 132 & 42 & 20 & 0 & 30 \\ 28 & 42 & 132 & 0 & 20 & 30 \\ 28 & 20 & 0 & 132 & 42 & 30 \\ 28 & 0 & 20 & 42 & 132 & 30 \\ 0 & 30 & 30 & 30 & 30 & 195 \end{pmatrix}, \tag{4.23}$$

and modified OPE coefficients

$$\begin{aligned}
\hat{b}^{j_1j_2j_3j_4}_{TTWW} &= -\frac{1}{2^{10}} \frac{1}{s^4_{j_1j_2} s^6_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_1(TTWW)} &= \frac{1}{2^8} \frac{c_{j_1j_2}}{s^5_{j_1j_2} s^6_{j_3j_4}}, \\
\hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_2(TTWW)} &= -\frac{1}{2^9} \frac{s_{j_1j_2j_3j_4}}{s^5_{j_1j_2} s^7_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_3(TTWW)} &= \hat{b}^{j_1j_2j_4j_3}_{\mathrm{I}_2(TTWW)}, \\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_1(TTWW)} &= -\frac{1}{2^{10}} \frac{9 - 8s^2_{j_1j_2}}{s^6_{j_1j_2} s^6_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_2(TTWW)} &= -\frac{3}{2^{10}} \frac{s^2_{j_1j_2j_3j_4}}{s^6_{j_1j_2} s^8_{j_3j_4}}, \\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_3(TTWW)} &= \hat{b}^{j_1j_2j_4j_3}_{\mathbf{II}_2(TTWW)}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_4(TTWW)} &= \hat{b}^{j_2j_1j_3j_4}_{\mathbf{II}_2(TTWW)}, \\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_5(TTWW)} &= \hat{b}^{j_2j_1j_4j_3}_{\mathbf{II}_2(TTWW)}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_6(TTWW)} &= -\frac{3}{2^{11}} \frac{13 - 12s^2_{j_3j_4}}{s^4_{j_1j_2} s^8_{j_3j_4}}.
\end{aligned} \tag{4.24}$$

For operators in class $WWWW$, we choose $C_K = \frac{c^2}{9}$ and we have

$$WWWW, \quad \beta_{WWWW} = 1, \quad \hat{b}^{j_1j_2j_3j_4}_{WWWW} = \frac{1}{2^{12}}\left(\frac{1}{s^6_{j_1j_2} s^6_{j_3j_4}} + \frac{1}{s^6_{j_1j_3} s^6_{j_2j_4}} + \frac{1}{s^6_{j_1j_4} s^6_{j_2j_3}}\right). \tag{4.25}$$

4.3 One-loop mutual information

Using the coefficients in the last subsection and the summation formulas in appendix B we can get the one-loop mutual information. The contributions from operators of different classes are respectively

$$
\begin{aligned}
I^{\text{1-loop}}_{WW} &= \frac{x^6}{12012}+\frac{x^7}{4290}+\frac{7x^8}{16830}+\frac{28x^9}{46189}+\frac{15x^{10}}{19019}+\frac{2x^{11}}{2093}+\frac{33x^{12}}{29900}+O(x^{13}),\\
I^{\text{1-loop}}_{\mathcal{U}\mathcal{U}} &= \frac{x^{10}}{3879876}+\frac{5x^{11}}{4056234}+\frac{33x^{12}}{9657700}+O(x^{13}),\\
I^{\text{1-loop}}_{TW\mathcal{U}} &= -\frac{x^{10}}{1939938}-\frac{5x^{11}}{2028117}-\frac{33x^{12}}{4828850}+O(x^{13}),\\
I^{\text{1-loop}}_{TTWW} &= \frac{x^{10}}{3879876}+\frac{5x^{11}}{4056234}+\frac{58x^{12}}{16900975}+O(x^{13}), \quad I^{\text{1-loop}}_{TW\mathcal{V}} = -\frac{x^{12}}{33801950}+O(x^{13}),\\
I^{\text{1-loop}}_{\mathcal{V}\mathcal{V}} &= I^{\text{1-loop}}_{\mathcal{E}\mathcal{E}} = \frac{x^{12}}{67603900}+O(x^{13}), \qquad I^{\text{1-loop}}_{TW\mathcal{X}} = O(x^{13}),\\
I^{\text{1-loop}}_{WW\mathcal{E}} &= -\frac{x^{12}}{33801950}+O(x^{13}), \qquad I^{\text{1-loop}}_{WWWW} = \frac{3163x^{12}}{1487285800}+O(x^{13}).
\end{aligned}
\tag{4.26}
$$

Summing them together, we get the additional contributions of the W_3 operator to one-loop mutual information

$$
I^{\text{1-loop}}_{(3)} = \frac{x^6}{12012}+\frac{x^7}{4290}+\frac{7x^8}{16830}+\frac{28x^9}{46189}+\frac{15x^{10}}{19019}+\frac{2x^{11}}{2093}+\frac{1644627x^{12}}{1487285800}+O(x^{13}), \tag{4.27}
$$

and this matches the gravity result in [28], i.e. $I^{\text{1-loop}}_{\text{spin-3}}$ in (2.14). Note that $I^{\text{1-loop}}_{WW}$ matches $I^{\text{1-loop}}_{(3)}$ to order x^{11}. We also find that there is cancellation

$$
I^{\text{1-loop}}_{\mathcal{U}\mathcal{U}}+I^{\text{1-loop}}_{TW\mathcal{U}}+I^{\text{1-loop}}_{TTWW}+I^{\text{1-loop}}_{TW\mathcal{V}}+I^{\text{1-loop}}_{\mathcal{V}\mathcal{V}}+I^{\text{1-loop}}_{TW\mathcal{X}} = O(x^{13}). \tag{4.28}
$$

5 W_4 operator

The case of CFT with $W(2,4)$ symmetry is similar to the CFT with $W(2,3)$ symmetry. In a CFT with $W(2,4)$ symmetry, there are operators W with conformal weights (4,0) and $\bar{W}$ with conformal weights (0,4), besides the stress tensor T and $\bar{T}$.

5.1 Construction of quasiprimary operators

The old holomorphic operators in the CFT with $W(2,4)$ symmetry are counted as

$$
\chi_{(2,4)} = \prod_{m=0}^{\infty}\frac{1}{1-x^{m+2}}\frac{1}{1-x^{m+4}}, \tag{5.1}
$$

among which the quasiprimary ones are counted as

$$
\begin{aligned}
(1-x)\chi_{(2,4)}+x = {} & 1+x^2+2x^4+3x^6+x^7+6x^8+3x^9+10x^{10}+7x^{11}+19x^{12}\\
& +14x^{13}+32x^{14}+O(x^{15}).
\end{aligned}
\tag{5.2}
$$

L_0	2	4	6	7	8	9	10	11	12	13	14	$\cdots$
#	1	2	3	1	6	3	10	7	19	14	32	$\cdots$
1	T	$\mathcal{A}$	$\mathcal{A}^{(6,m)}$		$\mathcal{A}^{(8,m)}$	$\mathcal{A}^{(9)}$	$\mathcal{A}^{(10,m)}$	$\mathcal{A}^{(11,m)}$	$\mathcal{A}^{(12,m)}$	$\mathcal{A}^{(13,m)}$	$\mathcal{A}^{(14,m)}$	$\cdots$
W		W	$\mathcal{U}$	$\mathcal{V}$	$\mathcal{X}$, $\mathcal{Z}$	$W^{(9,m)}$	$W^{(10,m)}$	$W^{(11,m)}$	$W^{(12,m)}$	$W^{(13,m)}$	$W^{(14,m)}$	$\cdots$
$\mathcal{E}$					$\mathcal{E}$		$\mathcal{E}^{(10)}$	$\mathcal{E}^{(11)}$	$\mathcal{E}^{(12,m)}$	$\mathcal{E}^{(13,m)}$	$\mathcal{E}^{(14,m)}$	$\cdots$
$\mathcal{F}$							$\mathcal{F}$		$\mathcal{F}^{(12)}$	$\mathcal{F}^{(13)}$	$\mathcal{F}^{(14,m)}$	$\cdots$
$\mathcal{G}$									$\mathcal{G}$		$\mathcal{G}^{(14)}$	$\cdots$
$\mathcal{H}$									$\mathcal{H}$		$\mathcal{H}^{(14)}$	$\cdots$
$\mathcal{I}$, $\mathcal{J}$											$\mathcal{I}$, $\mathcal{J}$	$\cdots$

Table 5. Old holographic nonidentity quasiprimary operators in the original CFT with $W(2,4)$ symmetry.

The nonidentity holomorphic primary operators are counted as

$$\frac{\chi_{(2,4)} - \chi_{(2)}}{\chi} = x^4 + x^8 + x^{10} + 2x^{12} + 2x^{14} + O(x^{15}), \tag{5.3}$$

with $\chi_{(2)}$ being defined in (3.1) and χ being defined in (4.4). At level 4, it is just W, at level 8 we denote it by $\mathcal{E}$, at level 10 we denote it by $\mathcal{F}$, at level 12 we denote them by $\mathcal{G}$ and $\mathcal{H}$, and at level 14 we denote them by $\mathcal{I}$ and $\mathcal{J}$. As usual we choose $\alpha_W = \frac{c}{4}$. In conformal family of W, at level 6 we have the quasiprimary operator

$$\mathcal{U} = (TW) - \frac{1}{6}\partial^2 W, \quad \alpha_{\mathcal{U}} = \frac{c(c+24)}{8}, \tag{5.4}$$

at level 7 we have the quasiprimary operator

$$\mathcal{V} = (T\mathrm{i}\partial W) - 2(\mathrm{i}TW) - \frac{1}{10}\mathrm{i}\partial^3 W, \quad \alpha_{\mathcal{V}} = \frac{3c(5c+22)}{5}, \tag{5.5}$$

at level 9 we have the two quasiprimary operators

$$\begin{aligned}
\mathcal{X} &= (\partial T \partial W) - \frac{2}{9}(T\partial^2 W) - \frac{4}{5}(\partial^2 T W) + \frac{1}{66}\partial^4 W, \\
\mathcal{Z} &= (T(TW)) - \frac{1}{3}(T\partial^2 W) - \frac{3}{10}(\partial^2 T W) + \frac{1}{44}\partial^4 W + \frac{273}{55c+137}\mathcal{X}, \\
\alpha_{\mathcal{X}} &= \frac{364c(55c+137)}{2475}, \quad \alpha_{\mathcal{Z}} = \frac{c(c+24)(c+31)(55c-6)}{8(55c+137)}.
\end{aligned} \tag{5.6}$$

Here $\mathcal{Z}$ is chosen such that the structure constant $C_{TW\mathcal{Z}} = 0$, and $\mathcal{X}$ is chosen such that it is orthogonal to $\mathcal{Z}$. The structure constants that will be useful are

$$C_{TW\mathcal{U}} = \frac{c(c+24)}{8}, \quad C_{TW\mathcal{V}} = -\frac{\mathrm{i}c(5c+22)}{5}, \quad C_{TW\mathcal{X}} = -\frac{2c(55c+137)}{55}. \tag{5.7}$$

To level 14 the old holomorphic quasiprimary operators are listed in table 5.

The additional new holomorphic quasiprimary operators in CFT^n are counted as

$$
\begin{aligned}
(1-x)(\chi^n_{(2,4)}-\chi^n_{(2)}) = {} & nx^4+n^2x^6+n^2x^7+\frac{n(n^2+4n+1)}{2}x^8+\frac{n(2n^2+3n-1)}{2}x^9 \\
& +\frac{n(n+1)(n^2+14n+3)}{6}x^{10}+\frac{n(n^3+7n^2+3n-1)}{2}x^{11} \\
& +\frac{n(n^4+36n^3+147n^2+84n+20)}{24}x^{12} \\
& +\frac{n(2n^4+35n^3+86n^2+19n-10)}{12}x^{13} \\
& +\frac{n(n^5+70n^4+695n^3+1310n^2+504n+60)}{120}x^{14}+O(x^{15}),
\end{aligned}
\tag{5.8}
$$

and they are listed in table 6.

5.2 Calculation of coefficients

The holomorphic quasiprimary operators that contribute to the one-loop mutual information are the ones in classes WW, $\mathcal{UU}$, $TW\mathcal{U}$, $TTWW$, $TW\mathcal{V}$, $\mathcal{VV}$, $TW\mathcal{X}$. For operators in classes WW, $\mathcal{UU}$ and $\mathcal{VV}$ we have

$$
I_{WW}=I_{\mathcal{OO}}|_{h=4}, \quad I_{\mathcal{UU}}=I_{\mathcal{OO}}|_{h=6}, \quad I_{\mathcal{VV}}=I_{\mathcal{OO}}|_{h=7}. \tag{5.9}
$$

For operators in class $TW\mathcal{U}$, we have

$$
\begin{aligned}
TW\mathcal{U}, \quad \mathrm{I}_1(TW\mathcal{U}) &= \mathrm{i}\partial TW\mathcal{U}-\frac{1}{2}T\mathrm{i}\partial W\mathcal{U}, \quad \mathrm{I}_2(TW\mathcal{U})=\mathrm{i}\partial TW\mathcal{U}-\frac{1}{3}TW\mathrm{i}\partial\mathcal{U}, \\
\mathbf{II}_1(TW\mathcal{U}) &= \partial T\partial W\mathcal{U}-\frac{4}{5}\partial^2TW\mathcal{U}-\frac{2}{9}T\partial^2W\mathcal{U}, \\
\mathbf{II}_2(TW\mathcal{U}) &= \partial TW\partial\mathcal{U}-\frac{6}{5}\partial^2TW\mathcal{U}-\frac{2}{13}TW\partial^2\mathcal{U}, \\
\mathbf{II}_3(TW\mathcal{U}) &= T\partial W\partial\mathcal{U}-\frac{2}{3}T\partial^2W\mathcal{U}-\frac{4}{13}TW\partial^2\mathcal{U},
\end{aligned}
\tag{5.10}
$$

the modified normalization factors

$$
\beta_{TW\mathcal{U}}=1, \quad \beta_{\mathrm{I}(TW\mathcal{U})}=\frac{2}{3}\begin{pmatrix} 9 & 6 \\ 6 & 8 \end{pmatrix}, \quad \beta_{\mathbf{II}(TW\mathcal{U})}=\frac{16}{585}\begin{pmatrix} 2366 & 1404 & 780 \\ 1404 & 4131 & 540 \\ 780 & 540 & 6930 \end{pmatrix}, \tag{5.11}
$$

and the modified OPE coefficients

$$
\begin{aligned}
\hat{b}^{j_1j_2j_3}_{TW\mathcal{U}} &= \frac{1}{2^{12}}\frac{1}{s^4_{j_1j_3}s^8_{j_2j_3}}, \qquad \hat{b}^{j_1j_2j_3}_{\mathrm{I}_1(TW\mathcal{U})}=\frac{1}{2^{11}}\frac{s_{j_1j_2}}{s^5_{j_1j_3}s^9_{j_2j_3}}, \\
\hat{b}^{j_1j_2j_3}_{\mathrm{I}_2(TW\mathcal{U})} &= \frac{1}{3\cdot 2^{11}}\frac{s_{j_1j_2}-3s_{j_1j_2j_3}}{s^5_{j_1j_3}s^9_{j_2j_3}}, \quad \hat{b}^{j_1j_2j_3}_{\mathbf{II}_1(TW\mathcal{U})}=\frac{1}{2^{10}}\frac{s^2_{j_1j_2}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}, \\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_2(TW\mathcal{U})} &= \frac{1}{13\cdot 2^{12}}\frac{10s^2_{j_1j_2}-52s^2_{j_1j_3}+65s^2_{j_2j_3}+78s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}, \\
\hat{b}^{j_1j_2j_3}_{\mathbf{II}_3(TW\mathcal{U})} &= \frac{1}{13\cdot 2^{11}}\frac{36s^2_{j_1j_2}+117s^2_{j_1j_3}-104s^2_{j_2j_3}+78s^2_{j_1j_2j_3}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}.
\end{aligned}
\tag{5.12}
$$

L_0	quasiprimary	???	#	#
4	W	×××	n	n
6	$\mathcal{U}$	×××	n	n^2
	TW	×××	n_2	
7	$\mathcal{V}$	×××	n	n^2
	n_2			
8	$\mathcal{X}$, $\mathcal{Z}$, $\mathcal{E}$	×××	$3n$	$\frac{n(n^2+4n+1)}{2}$
	$T\mathcal{U}$, $\mathcal{A}W$	×××	$2n_2$	
	WW	✓✓✓	$\frac{n_2}{2}$	
	TTW	×××	$\frac{n_3}{2}$	
	n_2			
9	$W^{(9,m)}$	×××	$2n$	$\frac{n(2n^2+3n-1)}{2}$
	$T\mathcal{V}$	×××	n_2	
	$\frac{n(2n^2+n-3)}{2}$			
10	$W^{(10,m)}$, $\mathcal{E}^{(10)}$, $\mathcal{F}$	×××	$6n$	$\frac{n(n+1)(n^2+14n+3)}{6}$
	$T\mathcal{X}$, $T\mathcal{Z}$, $T\mathcal{E}$, $\mathcal{AU}$, $\mathcal{A}^{(6,m)}W$	×××	$6n_2$	
	$W\mathcal{U}$	✓××	n_2	
	$TT\mathcal{U}$, $T\mathcal{A}W$	×××	$\frac{3n_3}{2}$	
	TWW	✓✓×	$\frac{n_3}{2}$	
	$TTTW$	×××	$\frac{n_4}{6}$	
	$\frac{3n(n^2-1)}{2}$			
11	$W^{(11,m)}$, $\mathcal{E}^{(11)}$	×××	$5n$	$\frac{n(n^3+7n^2+3n-1)}{2}$
	$TW^{(9,m)}$, $\mathcal{AV}$	×××	$3n_2$	
	$W\mathcal{V}$	✓××	n_2	
	$TT\mathcal{V}$	×××	$\frac{n_3}{2}$	
	$\frac{n(n-1)(n^2+7n+5)}{2}$			
12	$W^{(12,m)}$, $\mathcal{E}^{(12,m)}$ $\mathcal{F}^{(12)}$, $\mathcal{G}$, $\mathcal{H}$	×××	$12n$	$\frac{n(n^4+36n^3+147n^2+84n+20)}{24}$
	$TW^{(10,m)}$, $T\mathcal{E}^{(10)}$, $T\mathcal{F}$ $\mathcal{AX}$, $\mathcal{AZ}$, $\mathcal{AE}$, $\mathcal{A}^{(6,m)}\mathcal{U}$, $\mathcal{A}^{(8,m)}W$	×××	$14n_2$	
	$W\mathcal{X}$, $W\mathcal{Z}$	✓××	$2n_2$	
	$W\mathcal{E}$	×××	n_2	
	$\mathcal{UU}$	✓✓✓	$\frac{n_2}{2}$	
	$TT\mathcal{X}$, $TT\mathcal{Z}$, $TT\mathcal{E}$, $T\mathcal{AU}$, $T\mathcal{A}^{(6,m)}$W, $\mathcal{AA}W$	×××	$5n_3$	
	$TW\mathcal{U}$	✓✓✓	n_3	
	$\mathcal{A}WW$, WWW	✓✓×	$\frac{2n_3}{3}$	
	$TTT\mathcal{U}$, $TT\mathcal{A}W$	×××	$\frac{2n_4}{3}$	
	$TTWW$	✓✓✓	$\frac{n_4}{4}$	
	$TTTTW$	×××	$\frac{n_5}{24}$	
	$\frac{n(n-1)(2n^2+9n+5)}{2}$			
13	$W^{(13,m)}$, $\mathcal{E}^{(13,m)}$, $\mathcal{F}^{(13)}$	×××	$11n$	$\frac{n(2n^4+35n^3+86n^2+19n-10)}{12}$
	$TW^{(11,m)}$, $T\mathcal{E}^{(11)}$, $\mathcal{A}W^{(9,m)}$, $\mathcal{A}^{(6,m)}\mathcal{V}$, $\mathcal{A}^{(9)}W$,	×××	$10n_2$	
	$WW^{(9,m)}$, $\mathcal{UV}$	✓××	$3n_2$	
	$TTW^{(9,m)}$, $T\mathcal{AV}$	×××	$2n_3$	
	$TW\mathcal{V}$	✓✓✓	n_3	
	$TTT\mathcal{V}$	×××	$\frac{n_4}{6}$	
	$\frac{n(n-1)(2n^3+35n^2+97n+46)}{12}$			
14	$W^{(14,m)}$, $\mathcal{E}^{(14,m)}$, $\mathcal{F}^{(14,m)}$, $\mathcal{G}^{(14)}$, $\mathcal{H}^{(15)}$, $\mathcal{I}$, $\mathcal{J}$	×××	$22n$	$\frac{n(n^5+70n^4+695n^3+1310n^2+504n+60)}{120}$
	$TW^{(12,m)}$, $T\mathcal{E}^{(12,m)}$, $T\mathcal{F}^{(12)}$, $T\mathcal{G}$, $T\mathcal{H}$, $\mathcal{A}W^{(10,m)}$, $\mathcal{AE}^{(10)}$, $\mathcal{AF}$, $\mathcal{A}^{(6,m)}\mathcal{X}$, $\mathcal{A}^{(6,m)}\mathcal{Z}$, $\mathcal{A}^{(6,m)}\mathcal{E}$, $\mathcal{A}^{(8,m)}\mathcal{U}$, $\mathcal{A}^{(10,m)}W$	×××	$31n_2$	
	$WW^{(10,m)}$, $\mathcal{UX}$, $\mathcal{UZ}$	✓××	$6n_2$	
	$W\mathcal{E}^{(10)}$, $W\mathcal{F}$, $\mathcal{UE}$	×××	$3n_2$	
	$\mathcal{VV}$	✓✓✓	$\frac{n_2}{2}$	
	$TTW^{(10,m)}$, $TT\mathcal{E}^{(10)}$, $TT\mathcal{F}$, $T\mathcal{AX}$, $T\mathcal{AZ}$, $T\mathcal{AE}$, $T\mathcal{A}^{(6,m)}\mathcal{U}$, $T\mathcal{A}^{(8,m)}W$, $\mathcal{AAU}$, $\mathcal{AA}^{(6,m)}W$	×××	$\frac{27n_3}{2}$	
	$TW\mathcal{X}$	✓✓✓	n_3	
	$TW\mathcal{Z}$, $TW\mathcal{E}$	×××	$2n_3$	
	$T\mathcal{UU}$, $\mathcal{A}W\mathcal{U}$, $\mathcal{A}^{(6,m)}WW$	✓✓×	$\frac{5n_3}{2}$	
	$WW\mathcal{U}$	✓✓×	$\frac{n_3}{2}$	
	$TTT\mathcal{X}$, $TTT\mathcal{Z}$, $TTT\mathcal{E}$ $TT\mathcal{AU}$, $TT\mathcal{A}^{(6,m)}W$, $T\mathcal{AA}W$, $TTW\mathcal{U}$,	×××	$\frac{5n_4}{2}$	
	$T\mathcal{A}WW$, $TWWW$	✓✓×	$\frac{7n_4}{6}$	
	$TTTT\mathcal{U}$, $TTT\mathcal{A}W$	×××	$\frac{5n_5}{24}$	
	$TTTWW$	✓✓×	$\frac{n_5}{12}$	
	$TTTTTW$	×××	$\frac{n_6}{120}$	
	$\frac{n(n-1)(n+2)(5n^2+47n+24)}{12}$			
...	...	...	...	...

Table 6. Additional new holographic quasiprimary operators in CFT^n with $W(2,4)$ symmetry. The notations here are the same as the ones in table 4.

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In class $TW\mathcal{V}$, we have operators

$$TW\mathcal{V}, \quad \mathrm{I}_1(TW\mathcal{V}) = \mathrm{i}\partial TW\mathcal{V} - \frac{1}{2}T\mathrm{i}\partial W\mathcal{V}, \quad \mathrm{I}_2(TW\mathcal{V}) = \mathrm{i}\partial TW\mathcal{V} - \frac{2}{7}TW\mathrm{i}\partial\mathcal{V}, \tag{5.13}$$

the modified normalization factors

$$\beta_{TW\mathcal{V}} = 1, \quad \beta_{\mathrm{I}(TW\mathcal{V})} = \frac{2}{7}\begin{pmatrix} 21 & 14 \\ 14 & 18 \end{pmatrix}, \tag{5.14}$$

and the modified OPE coefficients

$$\begin{aligned}
\hat{b}^{j_1j_2j_3}_{TW\mathcal{V}} &= -\frac{\mathrm{i}}{2^{13}}\frac{s_{j_1j_2}}{s^5_{j_1j_3}s^9_{j_2j_3}}, \quad \hat{b}^{j_1j_2j_3}_{\mathrm{I}_1(TW\mathcal{V})} = -\frac{\mathrm{i}}{2^{15}}\frac{8s^2_{j_1j_2}+s^2_{j_1j_3}+2s^2_{j_2j_3}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}, \\
\hat{b}^{j_1j_2j_3}_{\mathrm{I}_2(TW\mathcal{V})} &= -\frac{\mathrm{i}}{7\cdot 2^{14}}\frac{10s^2_{j_1j_2}-28s^2_{j_1j_3}+35s^2_{j_2j_3}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}.
\end{aligned} \tag{5.15}$$

For operators in class $TW\mathcal{X}$, we have

$$TW\mathcal{X}, \quad \beta_{TW\mathcal{X}} = 1, \quad \hat{b}^{j_1j_2j_3}_{TW\mathcal{X}} = \frac{1}{2^{14}}\frac{s^2_{j_1j_2}}{s^6_{j_1j_3}s^{10}_{j_2j_3}}. \tag{5.16}$$

In class $TTWW$, we choose $C_K = \frac{c^2}{8}$ and we have operators

$$\begin{aligned}
TTWW, \quad \mathrm{I}_1(TTWW) &= \mathrm{i}\partial TTWW - T\mathrm{i}\partial TWW, \\
\mathrm{I}_2(TTWW) &= \mathrm{i}\partial TTWW - \frac{1}{2}TT\mathrm{i}\partial WW, \\
\mathrm{I}_3(TTWW) &= \mathrm{i}\partial TTWW - \frac{1}{2}TTW\mathrm{i}\partial W, \\
\mathbf{II}_1(TTWW) &= \partial T\partial TWW - \frac{2}{5}\partial^2 TTWW - \frac{2}{5}T\partial^2 TWW, \\
\mathbf{II}_2(TTWW) &= \partial TT\partial WW - \frac{4}{5}\partial^2 TTWW - \frac{2}{9}TT\partial^2 WW, \\
\mathbf{II}_3(TTWW) &= \partial TTW\partial W - \frac{4}{5}\partial^2 TTWW - \frac{2}{9}TTW\partial^2 W, \\
\mathbf{II}_4(TTWW) &= T\partial T\partial WW - \frac{4}{5}T\partial^2 TWW - \frac{2}{9}TT\partial^2 WW, \\
\mathbf{II}_5(TTWW) &= T\partial TW\partial W - \frac{4}{5}T\partial^2 TWW - \frac{2}{9}TTW\partial^2 W, \\
\mathbf{II}_6(TTWW) &= TT\partial W\partial W - \frac{4}{9}TT\partial^2 WW - \frac{4}{9}TTW\partial^2 W,
\end{aligned} \tag{5.17}$$

modified normalization factors

$$\beta_{TTWW} = 1, \quad \beta_{\mathrm{I}(TTWW)} = 2\begin{pmatrix} 4 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}, \quad \beta_{\mathbf{II}(TTWW)} = \frac{16}{45}\begin{pmatrix} 81 & 36 & 36 & 36 & 36 & 0 \\ 36 & 182 & 72 & 20 & 0 & 40 \\ 36 & 72 & 182 & 0 & 20 & 40 \\ 36 & 20 & 0 & 182 & 72 & 40 \\ 36 & 0 & 20 & 72 & 182 & 40 \\ 0 & 40 & 40 & 40 & 40 & 340 \end{pmatrix}, \tag{5.18}$$

and modified OPE coefficients

$$
\begin{aligned}
\hat{b}^{j_1j_2j_3j_4}_{TTWW} &= \frac{1}{2^{12}}\frac{1}{s^4_{j_1j_2}s^8_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_1(TTWW)} &= -\frac{1}{2^{10}}\frac{c_{j_1j_2}}{s^5_{j_1j_2}s^8_{j_3j_4}},\\
\hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_2(TTWW)} &= \frac{1}{2^{11}}\frac{s_{j_1j_2j_3j_4}}{s^5_{j_1j_2}s^9_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathrm{I}_3(TTWW)} &= \hat{b}^{j_1j_2j_4j_3}_{\mathrm{I}_2(TTWW)},\\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_1(TTWW)} &= \frac{1}{2^{12}}\frac{9-8s^2_{j_1j_2}}{s^6_{j_1j_2}s^8_{j_3j_4}}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_2(TTWW)} &= \frac{1}{2^{10}}\frac{s^2_{j_1j_2j_3j_4}}{s^6_{j_1j_2}s^{10}_{j_3j_4}},\\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_3(TTWW)} &= \hat{b}^{j_1j_2j_4j_3}_{\mathbf{II}_2(TTWW)}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_4(TTWW)} &= \hat{b}^{j_2j_1j_3j_4}_{\mathbf{II}_2(TTWW)},\\
\hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_5(TTWW)} &= \hat{b}^{j_2j_1j_4j_3}_{\mathbf{II}_2(TTWW)}, & \hat{b}^{j_1j_2j_3j_4}_{\mathbf{II}_6(TTWW)} &= \frac{1}{2^{11}}\frac{17-16s^2_{j_3j_4}}{s^4_{j_1j_2}s^{10}_{j_3j_4}}.
\end{aligned}
\tag{5.19}
$$

5.3 One-loop mutual information

The contributions from operators of different classes are respectively

$$
\begin{aligned}
I^{\text{1-loop}}_{WW} &= \frac{x^8}{218790}+\frac{4x^9}{230945}+\frac{3x^{10}}{76076}+\frac{5x^{11}}{71162}+\frac{11x^{12}}{101660}+\frac{11x^{13}}{72675}+\frac{1001x^{14}}{5058180}+O(x^{15}),\\
I^{\text{1-loop}}_{\mathcal{U}\mathcal{U}} &= \frac{x^{12}}{67603900}+\frac{x^{13}}{11700675}+\frac{13x^{14}}{46535256}+O(x^{15}),\\
I^{\text{1-loop}}_{TW\mathcal{U}} &= -\frac{x^{12}}{33801950}-\frac{2x^{13}}{11700675}-\frac{13x^{14}}{23267628}+O(x^{15}),\\
I^{\text{1-loop}}_{TTWW} &= \frac{x^{12}}{67603900}+\frac{x^{13}}{11700675}+\frac{163x^{14}}{581690700}+O(x^{15}),\\
I^{\text{1-loop}}_{TW\mathcal{V}} &= -\frac{x^{14}}{581690700}+O\left(x^{15}\right),\\
I^{\text{1-loop}}_{\mathcal{V}\mathcal{V}} &= \frac{x^{14}}{1163381400}+O(x^{15}), \qquad I^{\text{1-loop}}_{TW\mathcal{X}} = O(x^{15}).
\end{aligned}
\tag{5.20}
$$

Summing them together, we get the additional contributions of W_4 operator to one-loop mutual information

$$
I^{\text{1-loop}}_{(4)} = \frac{x^8}{218790}+\frac{4x^9}{230945}+\frac{3x^{10}}{76076}+\frac{5x^{11}}{71162}+\frac{11x^{12}}{101660}+\frac{11x^{13}}{72675}+\frac{1001x^{14}}{5058180}+O(x^{15}), \tag{5.21}
$$

and this matches the gravity result in [28], i.e. $I^{\text{1-loop}}_{\text{spin-4}}$ in (2.14). Note that there is cancellation

$$
I^{\text{1-loop}}_{\mathcal{U}\mathcal{U}}+I^{\text{1-loop}}_{TW\mathcal{U}}+I^{\text{1-loop}}_{TTWW}+I^{\text{1-loop}}_{TW\mathcal{V}}+I^{\text{1-loop}}_{\mathcal{V}\mathcal{V}}+I^{\text{1-loop}}_{TW\mathcal{X}} = O(x^{15}). \tag{5.22}
$$

6 Conclusion and discussion

In this paper we have calculated the one-loop entanglement entropy of two short intervals using OPE of twist operators in the CFT side. Following the strategy in [28] we took the $n \to 1$ limit of the Rényi entropy, and this allows us to get the one-loop entanglement entropy with higher order of the cross ratio x than before. We considered the contributions of stress tensor, W_3 operator, and W_4 operator. The results are in agreement with the

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ones of gravity side in [28]. It is notable that there are nontrivial cancellations in (4.28) and (5.22). We do not know if there may be some further indications for these cancellations.

In the gravity side, contributions of general spin-s fields to the entanglement entropy have been organized into different parts [28]. It would be nice to investigate if one can organize the CFT^n quasiprimary operators that appear in the OPE of twist operators so that some particular quasiprimary operators contribute to some particular parts of the entanglement entropy. It is expected that there are cancellations similar to (4.28) and (5.22) in contributions of a W_s operator with general s to the one-loop entanglement entropy.

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A Contributions of new quasiprimary operators with two old ones

In this appendix, we investigate the contributions of new holomorphic quasiprimary operators of CFT^n with two old holomorphic quasiprimary operators to the one-loop mutual information. We consider a general old holomorphic quasiprimary $\mathcal{O}$ with an integer conformal dimension $(h, 0)$. Using two of them we construct the new quasiprimary operators to order $2h + 6$,[2]

$$\begin{aligned}
&\mathcal{O}\mathcal{O}, \quad \mathrm{I}(\mathcal{O}\mathcal{O}) = \mathcal{O}\mathrm{i}\partial\mathcal{O} - \mathrm{i}\partial\mathcal{O}\mathcal{O}, \quad \mathbf{II}(\mathcal{O}\mathcal{O}) = \partial\mathcal{O}\partial\mathcal{O} - \frac{h}{2h+1}(\mathcal{O}\partial^2\mathcal{O} + \partial^2\mathcal{O}\mathcal{O}),\\
\mathbf{III}(\mathcal{O}\mathcal{O}) &= \mathrm{i}\partial\mathcal{O}\partial^2\mathcal{O} - \partial^2\mathcal{O}\partial\mathrm{i}\mathcal{O} - \frac{h}{3(h+1)}(\mathcal{O}\mathrm{i}\partial^3\mathcal{O} - \mathrm{i}\partial^3\mathcal{O}\mathcal{O}),\\
\mathbf{IV}(\mathcal{O}\mathcal{O}) &= \partial^2\mathcal{O}\partial^2\mathcal{O} - \frac{2h+1}{3(h+1)}(\partial\mathcal{O}\partial^3\mathcal{O} + \partial^3\mathcal{O}\partial\mathcal{O}) + \frac{h(2h+1)}{6(h+1)(2h+3)}(\mathcal{O}\partial^4\mathcal{O} + \partial^4\mathcal{O}\mathcal{O}),\\
\mathrm{V}(\mathcal{O}\mathcal{O}) &= \partial^2\mathcal{O}\mathrm{i}\partial^3\mathcal{O} - \mathrm{i}\partial^3\mathcal{O}\partial^2\mathcal{O} - \frac{2h+1}{2(2h+3)}(\mathrm{i}\partial\mathcal{O}\partial^4\mathcal{O} - \partial^4\mathcal{O}\mathrm{i}\partial\mathcal{O})\\
&\quad + \frac{h(2h+1)}{10(h+2)(2h+3)}(\mathcal{O}\mathrm{i}\partial^5\mathcal{O} - \mathrm{i}\partial^5\mathcal{O}\mathcal{O}),\\
\mathbf{VI}(\mathcal{O}\mathcal{O}) &= \partial^3\mathcal{O}\partial^3\mathcal{O} - \frac{3(h+1)}{2(2h+3)}(\partial^2\mathcal{O}\partial^4\mathcal{O} + \partial^4\mathcal{O}\partial^2\mathcal{O})\\
&\quad + \frac{3(h+1)(2h+1)}{10(h+2)(2h+3)}(\partial\mathcal{O}\partial^5\mathcal{O} + \partial^5\mathcal{O}\partial\mathcal{O})\\
&\quad - \frac{h(h+1)(2h+1)}{10(h+2)(2h+3)(2h+5)}(\mathcal{O}\partial^6\mathcal{O} + \partial^6\mathcal{O}\mathcal{O}).
\end{aligned} \tag{A.1}$$

Note that we have omitted the subscripts $j_1, j_2 = 0, 1, \cdots, n-1$ with $j_1 < j_2$, and so each equation above actually represent $\frac{n(n-1)}{2}$ operators.

[2] Some of the operators have been constructed in [27, 28], and the corresponding coefficients α_K and d_K have also been calculated therein.

The normalization of $\mathcal{O}$ is $\alpha_{\mathcal{O}}$, and for all these operators we choose $C_K = \alpha_{\mathcal{O}}$ and $\tilde{\alpha}_K = \alpha_{\mathcal{O}}^2$. Then we get the modified normalization factor

$$\begin{aligned}\beta_{\mathcal{O}\mathcal{O}} &= 1, \quad \beta_{\mathrm{I}(\mathcal{O}\mathcal{O})} = 4h, \quad \beta_{\mathbf{II}(\mathcal{O}\mathcal{O})} = \frac{4h^2(4h+1)}{2h+1}, \\ \beta_{\mathbf{III}(\mathcal{O}\mathcal{O})} &= \frac{16h^2(2h+1)(4h+3)}{3(h+1)}, \\ \beta_{\mathbf{IV}(\mathcal{O}\mathcal{O})} &= \frac{16h^2(2h+1)^2(4h+3)(4h+5)}{3(h+1)(2h+3)}, \\ \beta_{\mathrm{V}(\mathcal{O}\mathcal{O})} &= \frac{192h^2(h+1)(2h+1)^2(4h+5)(4h+7)}{5(h+2)(2h+3)}, \\ \beta_{\mathbf{VI}(\mathcal{O}\mathcal{O})} &= \frac{576h^2(h+1)^2(2h+1)^2(4h+5)(4h+7)(4h+9)}{5(h+2)(2h+3)(2h+5)}.\end{aligned} \tag{A.2}$$

We also have the modified OPE coefficients

$$\begin{aligned}\hat{b}^{j_1j_2}_{\mathcal{O}\mathcal{O}} &= \frac{1}{(2\mathrm{i})^{2h}}\frac{1}{s^{2h}_{j_1j_2}}, \quad \hat{b}^{j_1j_2}_{\mathrm{I}(\mathcal{O}\mathcal{O})} = \frac{2h}{(2\mathrm{i})^{2h}}\frac{c_{j_1j_2}}{s^{2h+1}_{j_1j_2}}, \quad \hat{b}^{j_1j_2}_{\mathbf{II}(\mathcal{O}\mathcal{O})} = \frac{h}{2(2\mathrm{i})^{2h}}\frac{(4h+1)-4hs^2_{j_1j_2}}{s^{2h+2}_{j_1j_2}}, \\ \hat{b}^{j_1j_2}_{\mathbf{III}(\mathcal{O}\mathcal{O})} &= \frac{h(2h+1)}{3(2\mathrm{i})^{2h}}\frac{c_{j_1j_2}\left[(4h+3)-4hs^2_{j_1j_2}\right]}{s^{2h+3}_{j_1j_2}}, \\ \hat{b}^{j_1j_2}_{\mathbf{IV}(\mathcal{O}\mathcal{O})} &= \frac{h(2h+1)}{12(2\mathrm{i})^{2h}}\frac{(4h+3)(4h+5)-4(2h+1)(4h+3)s^2_{j_1j_2}+8h(2h+1)s^4_{j_1j_2}}{s^{2h+4}_{j_1j_2}}, \\ \hat{b}^{j_1j_2}_{\mathrm{V}(\mathcal{O}\mathcal{O})} &= \frac{h(h+1)(2h+1)}{10(2\mathrm{i})^{2h}}\frac{c_{j_1j_2}\left[(4h+5)(4h+7)-4(2h+1)(4h+5)s^2_{j_1j_2}+8h(2h+1)s^4_{j_1j_2}\right]}{s^{2h+5}_{j_1j_2}}, \\ \hat{b}^{j_1j_2}_{\mathbf{VI}(\mathcal{O}\mathcal{O})} &= \frac{h(h+1)(2h+1)}{40(2\mathrm{i})^{2h}s^{2h+6}_{j_1j_2}}\Big[(4h+5)(4h+7)(4h+9)-12(h+1)(4h+5)(4h+7)s^2_{j_1j_2} \\ &\quad +24(h+1)(2h+1)(4h+5)s^4_{j_1j_2}-32h(h+1)(2h+1)s^6_{j_1j_2}\Big],\end{aligned} \tag{A.3}$$

with the definitions $s_{j_1j_2} = \sin(\frac{j_1-j_2}{n}\pi)$ and $c_{j_1j_2} = \cos(\frac{j_1-j_2}{n}\pi)$. Using (B.1), (B.2) and taking into the contributions of the antiholomorphic sector, we get the contributions of the above operators to the mutual information

$$\begin{aligned}I_{\mathcal{O}\mathcal{O}} = \frac{\Gamma(3/2)\Gamma(2h+1)}{\Gamma(2h+3/2)}\left(\frac{x}{4}\right)^{2h}\Bigg[&1+\frac{2h(2h+1)x}{4h+3}+\frac{(h+1)(2h+1)^2(4h+1)x^2}{2(16h^2+32h+15)} \\ &+\frac{(h+1)^2(2h+1)^2(2h+3)x^3}{3(16h^2+48h+35)}+\frac{(h+1)^2(h+2)(2h+1)(2h+3)^2x^4}{12(16h^2+64h+63)} \\ &+\frac{(h+1)^2(h+2)^2(2h+1)(2h+3)^2(2h+5)x^5}{30(64h^3+368h^2+636h+297)} \\ &+\frac{(h+1)(h+2)^2(h+3)(2h+1)(2h+3)^2(2h+5)^2x^6}{360(64h^3+432h^2+860h+429)}+O(x^7)\Bigg].\end{aligned} \tag{A.4}$$

Note that these operators only contribute to the one-loop part of the mutual information. In [28] there is the gravity result that for spin-s field one part of the one-loop entanglement entropy is

$$S^{(s)\text{1-loop}}_{\text{CDW,(I)}} = -\frac{\Gamma(3/2)\Gamma(2h+1)}{\Gamma(2h+3/2)}\left(\frac{x}{4}\right)^{2s} {}_3F_2(2s, 2s-1/2, 2s+1; 2s+3/2, 4s-1; x), \quad \text{(A.5)}$$

and our result (A.4) is in accord with this by setting $h = s$.

B Some summation formulas

We collect some useful summation formulas in this appendix. Firstly we define

$$f_m = \sum_{j=1}^{n-1} \frac{1}{\left(\sin\frac{\pi j}{n}\right)^{2m}}, \quad \text{(B.1)}$$

with m being an integer. As shown in [23], one has $f_m \sim n-1$ and

$$\tilde{f}_m = \lim_{n\to 1}\frac{f_m}{n-1} = \frac{\Gamma(3/2)\Gamma(m+1)}{\Gamma(m+3/2)}. \quad \text{(B.2)}$$

There are several summations that are related to (B.1), and these include

$$\begin{aligned}
&\sum_{j_1,j_2}^{\neq}\frac{1}{s^{2m}_{j_1j_2}} = nf_m, \qquad \sum_{j_1,j_2,j_3}^{\neq}\frac{1}{s^{2p}_{j_1j_2}s^{2q}_{j_1j_3}} = n(f_pf_q - f_{p+q}),\\
&\sum_{j_1,j_2,j_3}^{\neq}\frac{c_{j_1j_2}c_{j_1j_3}}{s^{2p+1}_{j_1j_2}s^{2q+1}_{j_1j_3}} = n(f_{p+q} - f_{p+q+1}) \\
&\sum_{j_1,j_2,j_3,j_4}^{\neq}\frac{1}{s^{2p}_{j_1j_2}s^{2q}_{j_3j_4}} = n(n-4)f_pf_q + 2nf_{p+q}.
\end{aligned} \quad \text{(B.3)}$$

All the summations indices in above equations are in the range $0 \le j_{1,2,3,4} \le n-1$. The first summation has the constraint $j_1 \neq j_2$, the second and third summations have the constraints $j_1 \neq j_2$, $j_1 \neq j_3$, and $j_2 \neq j_3$, and the last summation has the constraints $j_1 \neq j_2$, $j_1 \neq j_3$, $j_1 \neq j_4$, $j_2 \neq j_3$, $j_2 \neq j_4$, and $j_3 \neq j_4$. We use the same summation notations below.

We define that

$$\begin{aligned}
s_{p,q} &= \sum_{j_1,j_2,j_3}^{\neq}\frac{s^2_{j_1j_2}}{s^{2p}_{j_1j_3}s^{2q}_{j_2j_3}}, &\qquad t_{p,q} &= \sum_{j_1,j_2,j_3}^{\neq}\frac{s^4_{j_1j_2}}{s^{2p}_{j_1j_3}s^{2q}_{j_2j_3}},\\
u_{p,q} &= \sum_{j_1,j_2,j_3}^{\neq}\frac{s^2_{j_1j_2}c_{j_1j_3}c_{j_2j_3}}{s^{2p+1}_{j_1j_3}s^{2q+1}_{j_2j_3}}, &\qquad v_{p,q} &= \sum_{j_1,j_2,j_3}^{\neq}\frac{s_{j_1j_2}c_{j_1j_3}}{s^{2p}_{j_1j_3}s^{2q+1}_{j_2j_3}}.
\end{aligned} \quad \text{(B.4)}$$

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We have

$$
\begin{aligned}
s_{5,5} &= \frac{4n(n^2-1)^2(n^2-4)(n^2+11)(3n^4+10n^2+227)(5n^6+58n^4+325n^2+1052)}{6630710625}, \\
s_{5,6} &= \frac{2n(n^2-1)^2(n^2-4)}{99560120034375}(45553n^{14}+1328108n^{12}+19669231n^{10}+201786116n^8+1535925879n^6 \\
&\quad +8192615444n^4+29746589337n^2+64811480332), \\
s_{5,7} &= \frac{2n(n^2-1)^2(n^2-4)}{298680360103125}(13840n^{16}+448758n^{14}+7377133n^{12}+83185441n^{10}+691628526n^8 \\
&\quad +4541914744n^6+22337114089n^4+77828433057n^2+166234428412), \\
s_{5,8} &= \frac{n(n^2-1)^2(n^2-4)}{76163491826296875}(715083n^{18}+25534538n^{16}+461805573n^{14}+5699916578n^{12}+52320956483n^{10} \\
&\quad +375059702238n^8+2232468198983n^6+10411222626638n^4+35326719643878n^2+74499122340008), \\
s_{5,9} &= \frac{2n(n^2-1)^2(n^2-4)}{3038923323869245312 5}(14453970n^{20}+563655376n^{18}+11124486091n^{16}+149347794891n^{14} \\
&\quad +1500571528631n^{12}+11858181395071n^{10}+76527402573861n^8+425332156697681n^6 \\
&\quad +1912070720866171n^4+6372177472656981n^2+13322930703091276), \\
s_{5,10} &= \frac{2n(n^2-1)^2(n^2-4)}{4558384985803867968 75}(21967243n^{22}+928905348n^{20}+19867081060n^{18}+288349232835n^{16} \\
&\quad +3144899150355n^{14}+27151604038455n^{12}+192213812991645n^{10}+1148722121535645n^8 \qquad \text{(B.5)} \\
&\quad +6073947641495190n^6+26593077984745265n^4+87503665266114507n^2+181852852200342452), \\
s_{6,6} &= \frac{4n(n^2-1)^2(n^2-4)(2n^8+35n^6+321n^4+2125n^2+14797)}{59736072020625}(691n^8+10280n^6+75663n^4 \\
&\quad +355070n^2+1070296), \\
s_{6,7} &= \frac{2n(n^2-1)^2(n^2-4)}{407698691540765625}(1910462n^{18}+68101172n^{16}+1226741277n^{14}+14905903687n^{12} \\
&\quad +139242875522n^{10}+1046414082282n^8+6136429840777n^6+27331736137187n^4 \\
&\quad +89096568481962n^2+183491159525672), \\
s_{6,9} &= \frac{2n(n^2-1)^2(n^2-4)}{592590048154502835937 5}(285030529n^{22}+12033503724n^{20}+256501410985n^{18}+3681748010085n^{16} \\
&\quad +40260080243145n^{14}+353054399664045n^{12}+2550512661662865n^{10}+15681395452923615n^8 \\
&\quad +80490367507898250n^6+331149869650451675n^4+1032349661011754226n^2 \\
&\quad +2076990250139446856), \\
s_{6,10} &= \frac{2n(n^2-1)^2(n^2-4)}{1368883011236901551015625}(6671146880n^{24}+303588222092n^{22}+6973032426942n^{20} \\
&\quad +107805919807535n^{18}+1267311092051085n^{16}+11977039037640765n^{14}+93701187618319965n^{12} \\
&\quad +619839214947433575n^{10}+3572385606811692975n^8+17593549852111719955n^6 \\
&\quad +70678251505956283825n^4+217481207149114176078n^2+434593017048771078328),
\end{aligned}
$$

$$
\begin{aligned}
t_{6,6} &= \frac{4n(n^2-1)^2(n^2-4)}{99560120034375}(91053n^{14}+1751258n^{12}+15802641n^{10}+86621976n^8+122885159n^6 \\
&\quad -1216809126n^4-8233724853n^2-22129450108), \\
t_{6,7} &= \frac{2n(n^2-1)^2(n^2-4)}{298680360103125}(55300n^{16}+1244154n^{14}+13403599n^{12}+89917783n^{10}+353556798n^8 \\
&\quad +63068872n^6-7097114693n^4-39444148809n^2-100293199004), \qquad \text{(B.6)} \\
t_{6,8} &= \frac{n(n^2-1)^2(n^2-4)}{6930877756193015625}(259976261n^{18}+6702162006n^{16}+83850278121n^{14}+668341854016n^{12} \\
&\quad +3453872052321n^{10}+10488470468166n^8-12276550624049n^6-301684860616404n^4 \\
&\quad -1511339984382654n^2-3720039665967784),
\end{aligned}
$$

$$
\begin{aligned}
t_{6,10}=&\frac{2n(n^2-1)^2(n^2-4)}{5925900481545028359375}(1140875688n^{22}+36914466948n^{20}+588438944965n^{18}+6119943169290n^{16}\\
&+45084693422310n^{14}+239240140814910n^{12}+866020397951400n^{10}+1596578940865050n^{8}\\
&-9709837053108030n^{6}-105197092471166730n^{4}-475582750715286333n^{2}-1131502803657349468),
\end{aligned}
$$

$$
\begin{aligned}
u_{5,5}=&-\frac{8n(n^2-1)^2(n^2-4)^2(5n^6+58n^4+325n^2+1052)^2}{218813450625},\\
u_{5,6}=&-\frac{8n(n^2-1)^2(n^2-4)^2(5n^6+58n^4+325n^2+1052)(691n^8+10280n^6+75663n^4+355070n^2+1070296)}{298680360103125},\\
u_{5,7}=&-\frac{16n(n^2-1)^2(n^2-4)^2(5n^6+58n^4+325n^2+1052)}{896041080309375}(105n^{10}+1907n^8+17305n^6+102921n^4\\
&+436090n^2+1256072),\\
u_{5,9}=&-\frac{8n(n^2-1)^2(n^2-4)^2(5n^6+58n^4+325n^2+1052)}{91167699716077359375}(219335n^{14}+5426224n^{12}+67562250n^{10}\\
&+561172268n^8+3465459895n^6+16695492492n^4+63127741520n^2+172125054016),
\end{aligned}
\tag{B.7}
$$

$$
\begin{aligned}
v_{5,6}=&-\frac{2n(n^2-1)^2(n^2-4)(5n^6+58n^4+325n^2+1052)}{298680360103125}(1382n^{10}+28682n^8+307961n^6+2295661n^4\\
&+13803157n^2+92427157),\\
v_{5,8}=&-\frac{2n(n^2-1)^2(n^2-4)(5n^6+58n^4+325n^2+1052)}{228490475478890625}(10851n^{14}+296451n^{12}+4149467n^{10}\\
&+39686267n^8+292184513n^6+1777658113n^4+9611679169n^2+61430943169),\\
v_{7,4}=&-\frac{4n(n^2-1)^2(n^2-4)(n^2+11)(3n^4+10n^2+227)}{27152760009375}(105n^{10}+1907n^8+17305n^6+102921n^4\\
&+436090n^2+1256072),\\
v_{9,4}=&-\frac{2n(n^2-1)^2(n^2-4)(n^2+11)(3n^4+10n^2+227)}{2762657567153859375}(219335n^{14}+5426224n^{12}+67562250n^{10}\\
&+561172268n^8+3465459895n^6+16695492492n^4+63127741520n^2+172125054016).
\end{aligned}
\tag{B.8}
$$

We define

$$
\begin{aligned}
a_{p,q,r,s}&=\sum_{j_1,j_2,j_3,j_4}^{\neq}\frac{1}{s_{j_1j_2}^{2p}s_{j_3j_4}^{2q}s_{j_1j_3}^{2r}s_{j_2j_4}^{2s}},\\
b_{p,q,r,s}&=\sum_{j_1,j_2,j_3,j_4}^{\neq}\frac{c_{j_1j_2}c_{j_1j_3}}{s_{j_1j_2}^{2p+1}s_{j_3j_4}^{2q}s_{j_1j_3}^{2r+1}s_{j_2j_4}^{2s}},\\
c_{p,q,r,s}&=\sum_{j_1,j_2,j_3,j_4}^{\neq}\frac{c_{j_1j_2}c_{j_3j_4}c_{j_1j_3}c_{j_2j_4}}{s_{j_1j_2}^{2p+1}s_{j_3j_4}^{2q+1}s_{j_1j_3}^{2r+1}s_{j_2j_4}^{2s+1}},\\
d_{p,q,r,s}&=\sum_{j_1,j_2,j_3,j_4}^{\neq}\frac{c_{j_1j_2}c_{j_3j_4}}{s_{j_1j_2}^{2p+1}s_{j_3j_4}^{2q+1}s_{j_1j_3}^{2r}s_{j_2j_4}^{2s}}.
\end{aligned}
\tag{B.9}
$$

We have

$$
\begin{aligned}
a_{2,2,2,2}=&\frac{4n(n^2-1)(n^2-4)(n^2-9)}{54273594375}(21n^{10}+1994n^8+105648n^6+4785522n^4+141534331n^2+2127620484),\\
a_{2,2,2,3}=&\frac{2n(n^2-1)(n^2-4)(n^2-9)}{194896477400625}(11120n^{12}+1096256n^{10}+59129609n^8+2551249273n^6+80209669687n^4\\
&+1740731207971n^2+22574176404084),
\end{aligned}
$$

$$a_{2,3,2,3} = \frac{2n(n^2-1)(n^2-4)(n^2-9)}{32157918771103125}(118663n^{14}+12334362n^{12}+688418766n^{10}+28724959384n^{8} \tag{B.10}$$
$$+988368726279n^{6}+26290949481846n^{4}+506319939369292n^{2}+6044979290991408),$$
$$a_{3,3,3,3} = \frac{2n(n^2-1)(n^2-4)(n^2-9)}{302879357945634782812 5}(23125517n^{18}+3014718238n^{16}+205212344463n^{14}+9781329558932n^{12}$$
$$+371907750192979n^{10}+12650688547612206n^{8}+368606252765781785n^{6}$$
$$+9113476333879452640n^{4}+174930349600869335256n^{2}+2059147054618331397984),$$

$$b_{2,2,2,2} = -\frac{2n(n^2-1)(n^2-4)(n^2-9)}{194896477400625}(3755n^{12}+386267n^{10}+21570773n^{8}+929319721n^{6}+32134979884n^{4}$$
$$+854833946512n^{2}+12005931953088), \tag{B.11}$$

$$c_{2,2,2,2} = -\frac{2n(n^2-1)(n^2-4)(n^2-9)}{6431583754220625}(9401n^{14}+529374n^{12}+10085796n^{10}-183846586n^{8}-21275167485n^{6}$$
$$-1024689595860n^{4}-31153950441712n^{2}-468739784852928), \tag{B.12}$$

$$d_{2,2,2,2} = -\frac{2n(n^2-1)(n^2-4)(n^2-9)}{64965492466875}(510n^{12}+75236n^{10}+5064199n^{8}+233577843n^{6}+9882627307n^{4}$$
$$+280124377421n^{2}+3986367653484), \tag{B.13}$$
$$d_{2,2,2,3} = -\frac{2n(n^2-1)(n^2-4)(n^2-9)}{1531329465290625}(283n^{14}+84812n^{12}+7204196n^{10}+371271219n^{8}+15346533334n^{6}$$
$$+519473950801n^{4}+12716870657687n^{2}+170109742777668).$$

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